Final Exam in Algorithms and Data Structures 1 (1DL210)

Department of Information Technology
Uppsala University
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Location: Ekonomikum, B:154
Time: 08:00 - 13:00
No books or calculator allowed

Directions:

1. Do not write on the back of the paper
2. Write your name on each sheet of paper
3. **Important** Unless explicitly stated otherwise, justify you answer carefully! Answers without justification do not give any credits.

Good Luck!

Problem 1 (10p)

Order these functions in order of asymptotic growth rate, with the most rapidly growing first. If two of them have the same asymptotic growth rate, state that fact. No justification is needed.

$\lg(n^2)$ $\quad n^{1000}$ $\quad \lg n$ $\quad 10^{-10}\cdot 2^n$ $\quad 10^{-1000}\cdot 2^{n^2}$ $\quad 4^n$ $\quad n \lg n$ $\quad \lg(2^n)$

Problem 2 (10p)

Consider the algorithm Selection-Sort, given below.

```
SELECTION-SORT(A)
n ← length[A]
for j ← 1 to n − 1
    do smallest ← j
       for i ← j + 1 to n
           do if A[i] < A[smallest]
                 then smallest ← i
       exchange A[j] ↔ A[smallest]
```

a) What is the worst case asymptotic running time of SELECTION-SORT?

b) What is the best case asymptotic running time of SELECTION-SORT?

Justify your answers in at most 3 lines.

Problem 3 (15p)

Show all subarrays created, and the relationship between them, when executing merge sort on the array [3,5,8,2,6,13,9,4]. No justification needed.

Problem 4 (10p) Assume that we have a max-heap H and an integer x such that $x <$ HEAP-MAXIMUM(H). Assume also that we construct the heaps $H_1, \ldots, H_4$ in the following way:

- Let H_1 be the resulting heap after executing MAX-HEAP-INSERT(H, x).
- Let H_2 be the result of executing HEAP-EXTRACT-MAX(H_1).
- Let H_3 be the resulting heap after executing HEAP-EXTRACT-MAX(H).
- Let H_4 be the result of executing MAX-HEAP-INSERT(H_3, x).

Is it always the case that $H_2 = H_4$? In other words, does it matter in which order we do MAX-HEAP-INSERT and HEAP-EXTRACT-MAX, as long as we do not remove the value we just inserted? If your answer is yes, justify in no more than 5 lines. If your answer is no, give a counterexample by providing concrete values for H and x and drawing H_1, H_2, H_3 and H_4.

Problem 5 (10p)

Assume you have the set $S = \{1, 8, 15, 22, 29, 36\}$ and you want to insert them into a hash table T of size at most 10, using chaining to resolve collisions.

a) Provide a size of T and a suitable hash function h, such that

  - the distribution of elements in T by using h would be good for random input
  - h performs badly for the elements in S

b) Provide a size of T and a suitable hash function h, such that

  - the distribution of elements in T by using h would be good for random input
  - h performs well for the elements in S

Justify each answer in at most 3 lines.

Problem 6 (15p)

a) For each of the sequences below, determine if it is a possible sequence of values that we could discover in a search in a binary search tree when searching for a value v. If yes, provide a suitable value for v. If not, explain why in 3 lines.

   i) 100, 50, 2, 45, 20, 1

   ii) 100, 50, 2, 45, 20, 3

b) Give a possible sequence of discovered values, of length 8, when searching in a binary search tree for the value 42. No justification needed.

Problem 7 (5p)

What is the *maximal* height (i.e. the maximum number of levels) of a binary search tree with n elements? Justify your answer in at most 3 lines.

Problem 8 (15p)

Give one possible BFS traversal (i.e. a sequence of nodes) starting from node A of the graph shown in Figure 1. Print the nodes *only* when they are finished. No justification needed.

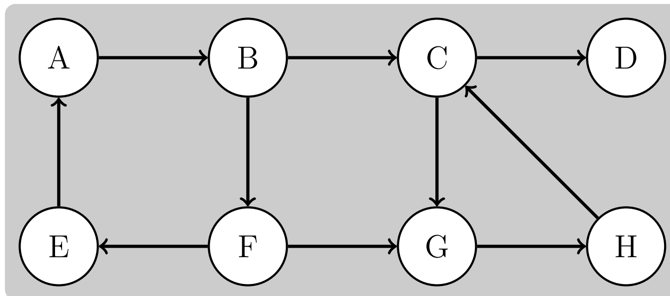

Figure 1: A graph

Problem 9 (10p)

The *complete graph* with n nodes, written K_n, is the graph with n nodes that has an edge between every pair nodes. In other words, in K_n every node is reachable from every other node. In how many different ways can the nodes be discovered (marked red) if we execute DFS(K_n)? Justify your answer in at most 5 lines.