Final Exam in Algorithms and Data Structures 1 (1DL210)

Department of Information Technology
Uppsala University
2013–10–18
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Location: Polacksbacken
Time: 08:00 - 13:00
No books or calculator allowed

Directions:

1. Do not write on the back of the paper
2. Write your anonymous code on each sheet of paper
3. **Important** Unless explicitly stated otherwise, justify you answer carefully! Answers without justification do not give any credits.

Good Luck!

Problem 1 Indicate whether the following statements are true or false. No justification is needed, and each question worths 2 points.

a) $2n + 13 = \Theta(n)$

c) a heap can not be both min-heap and a max-heap at the same time

d) $2^n = \Omega(3^n)$

e) $2n = \Omega(3n)$

f) the worst case running time of Insertion Sort is $O(2^n)$

Solution 1 The answers are:

a) true

c) false.

d) false

e) true

f) true

Problem 2 What is the worst-case running time of MEGESORT in the following cases:

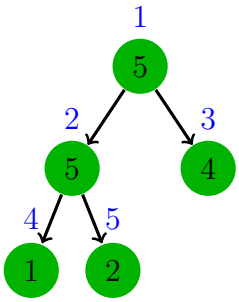
- if all the elements are equal.
- if the elements are increasing from left to right.

Solution 2 The time complexity of Mergesort is $\Theta(nlog(n))$ in all the cases.

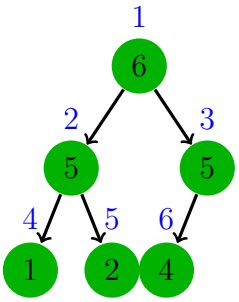
Problem 3 Suppose that we first insert an element x into a heap that does not already contain x. Suppose that we now delete x from the heap. Will the new heap be identical to the original one? If *yes* give the reason in no more than 3 lines. If *no* give a counter-example.

Solution 3 Assume that the heap is a max-heap. The answer should be no. Here is a counter-example:

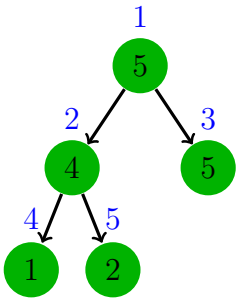
Insert 6 in the following heap:



The resulting heap is the following:



Then deleting 6 from the heap will result in the following heap:

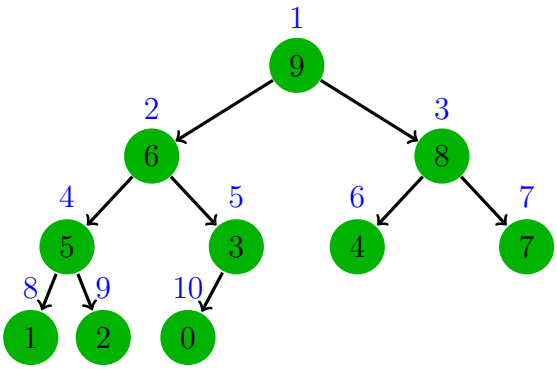


Problem 4 Give the max-heap that results when the keys

`1 9 4 3 5 8 7 6 2 0`

are inserted from left to right into an initially empty max-heap.

Solution 4 Below the resulting heap:

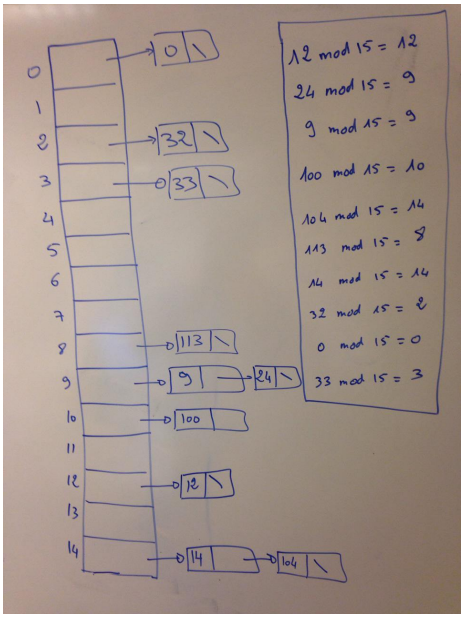


Problem 5 Consider inserting the following keys into a hash table of length $m = 15$, in the order they are listed (first 12, then 24, and so on):

$$12 \;\; 24 \;\; 9 \;\; 100 \;\; 104 \;\; 113 \;\; 14 \;\; 32 \;\; 0 \;\; 33$$

The auxiliary hash function is given by $(k \mod m)$. Draw the resulting hash table if we use chaining to resolve collisions.

Solution 5 Here you should state explicitly if you consider the first index of your array is 0 or 1. Let us assume that the first index of the array is 0. Then the resulting hash table is the following:



Problem 6 Consider a hash table H of a given size $n > 0$. Does increasing the size of H to $3n$ necessarily imply that the probability of collisions decreases by approximately one third? (Your answer should not be longer than **three lines**).

Solution 6 A similar answer can be find at: `https://www.it.uu.se/edu/course/homepage/algdstr1/ht10/extentalosn.pdf`

Problem 7 Suppose that we have numbers between 1 and 100 in a binary search tree and want to search for the number 45. Which of the following sequences could possibly be the sequence of nodes examined?

- 25, 2, 14, 10, 39, 34, 77, 63.
- 1, 2, 3, 5, 6, 7, 18, 29.
- 59, 48, 36, 5, 4, 3, 2, 1.
- 2, 9, 37, 19, 36, 82, 31, 78, 63.

Solution 7 Here is a possible solution:

- "25, 2, 14, 10, 39, 34, 77, 63" is not a possible sequence to investigate during during the search for 45 in a binary search tree. The first element in the sequence must be the root element. When we compare the root key 25 with the sought key 45, we see that 25 is less than 45, and so 45, if present in the tree, must be in the right sub-tree of the root node. The second element in the sequence (2) should therefore be the root of the right subtree of the root node. But all keys in the right subtree are strictly greater than 25, and so 2 cannot be the key root of the right subtree.

- "1, 2, 3, 5, 6, 7, 18, 29" is a possible sequence to investigate. It is the sequence that is investigated in the linear chain tree consisting of elements $\{1, 2, 3, 5, 6, 7, 18, 29\}$, where no node has a left child.

- "59, 48, 36, 5, 4, 3, 2, 1" is not a possible sequence to investigate. The reason is similar to the first sequence since we can't go from 36 to 5.

- "2, 9, 37, 19, 36, 82, 31, 78, 63" is not a possible sequence to investigate. The reason is similar to the first sequence since we can't go from 37 to 19.

Problem 8 Is the operation of deletion in a binary search tree commutative in the sense that deleting x and then y from a binary search tree leaves the same tree as deleting y and then x? Argue why it is so (no more than fives lines) or give a counter-example.

Solution 8 The answer is no. Below a counter example where $x = 6$ and $y = 5$