

Time: 8.00-13.00. Limits for the credits 3, 4, 5 are 18, 25 and 32 points, respectively. The solutions should be well motivated.

Permitted aids: The course book or copies thereof. Hand-written sheet of formulae. Pocket calculator. Dictionary. *No electronic device with internet connection.*

1. Let $\{w_t\}$, $t = 0, 1, 2, \dots$ be a Gaussian white noise process with $\text{var}(w_t) = 4$ and let

$$x_t = 1.2 + 0.4w_t w_{t-2} + 0.1w_{t-2} w_{t-4}.$$

Calculate the mean and autocovariance function of x_t and state whether it is weakly stationary. (5p)

2. For the ARMA(p, q) models below, where $\{w_t\}$ are white noise processes, find p and q and determine whether they are causal and/or invertible. (6p)

- (a) $x_t = w_t - 0.5w_{t-1}$
- (b) $x_t = 0.16x_{t-2} + w_t - 0.5w_{t-1}$
- (c) $x_t = x_{t-1} + w_t - w_{t-1}$
- (d) $x_t = -0.3x_{t-1} + 0.1x_{t-2} + w_t$

3. Let $\{w_t\}$ be a white noise process with variance $\sigma_w^2 = 1$ and define x_t through

$$x_t = w_t + \frac{1}{2}w_{t-1} - \frac{1}{4}w_{t-2} - \frac{1}{8}w_{t-3}.$$

Calculate the autocorrelation function $\rho(h)$ for $h = 1, 2, 3, 4, 5, 6$. (5p)

4. Consider the process

$$x_t = -0.5x_{t-1} + w_t - 0.9w_{t-1} + 0.2w_{t-2},$$

where $\{w_t\}$ is normally distributed white noise with variance $\sigma_w^2 = 0.1$. We observe x_t up to time $t = 100$, where the last four observations are $x_{97} = 0.4$, $x_{98} = 0.3$, $x_{99} = 0.2$ and $x_{100} = 0.1$.

- (a) Predict the values of x_{101} and x_{102} . Approximations are permitted. (4p)
- (b) Calculate 95% prediction intervals for x_{101} and x_{102} . (3p)

5. A time series $\{x_t\}$ follows the model

$$x_t = 0.5x_{t-1} + w_t,$$

where $\{w_t\}$ is normally distributed white noise with variance $\sigma_w^2 = 2$. This series is used as input for constructing

$$y_t = 0.6y_{t-1} + x_t,$$

for $t = 0, \pm 1, \pm 2, \dots$

(a) Calculate the spectral density of x_t at the frequencies $\omega = 0.1$ and $\omega = 0.4$. (2p)

(b) Calculate the spectral density of y_t at the frequencies $\omega = 0.1$ and $\omega = 0.4$. (3p)

(c) Compare and discuss your results. (1p)

6. Four time series with $n = 100$ observations each were simulated according to the models

$$x_t = 0.9x_{t-1} + w_t, \quad (1)$$

$$x_t = -0.9x_{t-1} + w_t, \quad (2)$$

$$x_t = 0.9x_{t-4} + w_t, \quad (3)$$

$$x_t = 0.9x_{t-12} + w_t, \quad (4)$$

where $\{w_t\}$ was normally distributed white noise with variance $\sigma_w^2 = 1$.

For three of these series, the corresponding spectral densities were estimated (nonparametric, spans=4), and they are depicted in figures 1-3.

Match three of the models (1)-(4) with the figures 1-3. Motivate your answer. (5p)

7. Consider the system

$$x_{1,t} = x_{1,t-1} + w_{1,t},$$

$$x_{2,t} = 0.8x_{1,t-1} + w_{2,t},$$

where $\{w_{1,t}\}$ and $\{w_{2,t}\}$ are independent white noise processes.

(a) Argue that $x_{1,t}$ and $x_{2,t}$ are not stationary. (3p)

(b) Find a constant a such that $x_{1,t} - ax_{2,t}$ is stationary. (3p)

GOOD LUCK!

Appendix: figures

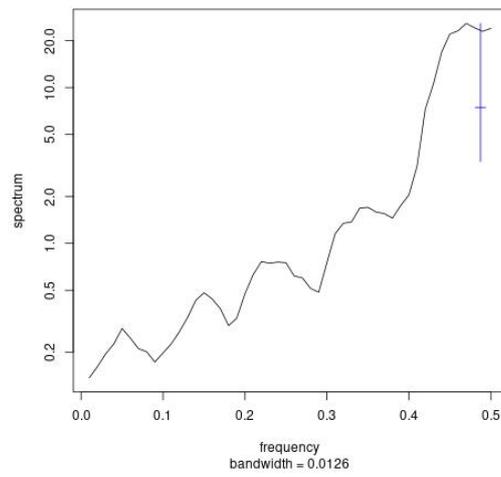


Figure 1: Estimated spectral density, problem 6.

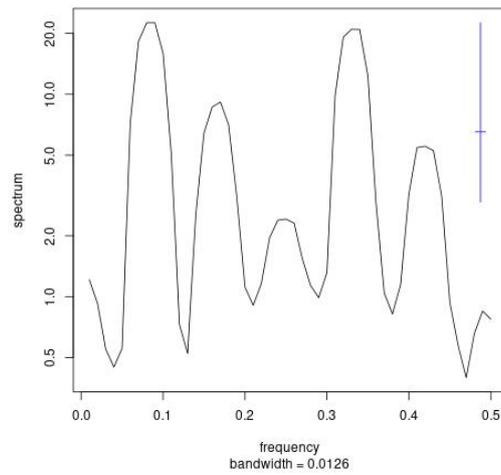


Figure 2: Estimated spectral density, problem 6.

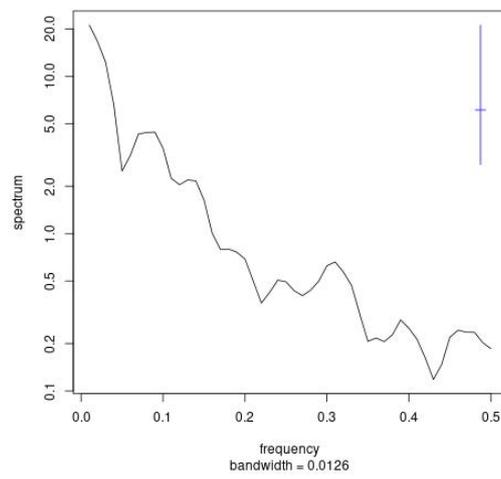


Figure 3: Estimated spectral density, problem 6.