SVAR

BASKURSTENTA
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LÖSNINGAR P

(1) $\frac{1-x}{2}$ (2) $1/2$ (3) 10.000 (4) $\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

(5) $-3/2$ (6) 4 och 2. (7) $-7/9$ (8) $x=-1$ eller 2

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(9) $$\frac{2x+1}{x}>1 \iff \frac{2x+1}{x}-1>0 \iff \frac{2x+1-x}{x}>0 \iff$$

$$\boxed{\frac{x+1}{x}>0}.$$

x		-1		0	
$x+1$	$- - -$	0	$+ + + +$		$+$
x			$- - - - -$	0	$+ + +$
$\frac{x+1}{x}$	$+ + +$ ↑	0	$- -$	$*$	$+ + +$ ↑

Svar: $\boxed{x>0 \text{ eller } x<-1}$

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(10) Basfall: $VL(n=0)=5^0=1 \quad HL(n=0)=\frac{5^1-1}{4}=1$ OK

Ind. steg: Visa $\underbrace{\sum_{k=0}^{n}5^k=\frac{5^{n+1}-1}{4}}_{A} \Rightarrow \underbrace{\sum_{k=0}^{n+1}5^k=\frac{5^{(n+1)+1}-1}{4}}_{B}$

(A) samt!

$$VL(B)=\sum_{k=0}^{n+1}5^k=\sum_{k=0}^{n}5^k+5^{n+1}=\frac{5^{n+1}-1}{4}+5^{n+1}=\frac{5^{n+1}-1+4\cdot5^{n+1}}{4}=$$

$$=\frac{5^{n+1}+4\cdot5^{n+1}-1}{4}=\frac{5^{n+1}\overbrace{(1+4)}^{5}-1}{4}=\frac{5^{n+2}-1}{4}=HL(B)$$

Så induktionsteget OK och då ger induktionsaxiomet att resultatet är sant. $\square$

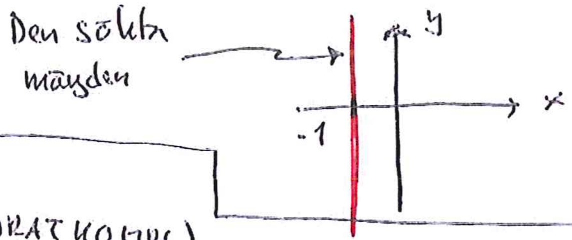
(11) Hanen kan bara väljas ut på ett sätt men de två honorna på $\binom{10}{2} = \frac{10 \cdot 9}{2} = \underline{45}$ sätt – det handlar ju om delmängden av 2 element valda ur 10.

Svar: $\boxed{45 \text{ sätt}}$

(12) $Z = x + iy \Rightarrow \operatorname{Im}(iz + 1 + 2i) = \operatorname{Im}(i(x + iy) + 1 + 2i) =$

$= \operatorname{Im}(ix - y + 1 + 2i) = \operatorname{Im}((1-y) + i(2+x)) = \underline{2+x}$

Så $\operatorname{Im} = 1 \Leftrightarrow 2 + x = 1 \Leftrightarrow \underline{x = -1,\ y \text{ godtycklig}}$



(13) $x^2 - 4x + y^2 + 6y = 3 \Leftrightarrow$ (KVADRATKOMPL.)

$(x^2 - 4x + 4) - 4 + (y^2 + 6y + 9) - 9 = 3 \Leftrightarrow \underline{(x-2)^2 + (y+3)^2 = 16}$

Svar: $\boxed{\text{Medelpunkt } (2, -3), \text{ radie } 4}$

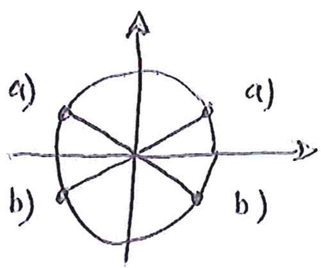
(14) $\underline{\underline{a^{\lg b}}} = 10^{\lg(a^{\lg b})} = 10^{\lg b \cdot \lg a} = 10^{\lg a \cdot \lg b} = 10^{\lg(b^{\lg a})} = \underline{\underline{b^{\lg a}}}$

(15) $\cos^2 x \quad 3\sin^2 x = 0 \overset{\text{TRIG. ETTAN}}{\Leftrightarrow} (1 - \sin^2 x) - 3\sin^2 x = 0 \Leftrightarrow$

$\sin^2 x = \frac{1}{4} \Rightarrow \underline{\sin x = \pm 1/2}$.

a) $\sin x = \frac{1}{2} \Rightarrow \begin{cases} x = \boxed{\frac{\pi}{6} + 2n\pi} \\ x = \pi - \frac{\pi}{6} + 2n\pi = \boxed{\frac{5\pi}{6} + 2n\pi} \end{cases}$

b) $\sin x = -\frac{1}{2} \Rightarrow \begin{cases} x = \boxed{-\frac{\pi}{6} + 2n\pi} \\ x = \pi - \left(-\frac{\pi}{6}\right) + 2n\pi = \boxed{\frac{7\pi}{6} + 2n\pi} \end{cases}$



(16) $z^4 = -16.$

Skriv på polär form:

$$\begin{cases} z = r(\cos\theta + i\sin\theta) \Rightarrow z^4 = r^4(\cos 4\theta + i\sin 4\theta) \\ -16 = 16(-1+0\cdot i) = 16(\cos\pi + i\sin\pi) \end{cases}$$

Ekvationen blir: $(\cos 4\theta + i\sin 4\theta) = 16(\cos\pi + i\sin\pi) \Leftrightarrow$

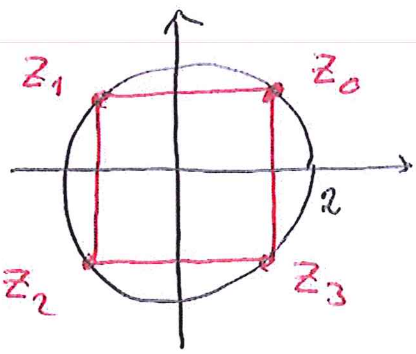
$$\begin{cases} r^4 = 16 \\ 4\theta = \pi + 2k\pi \end{cases} \Leftrightarrow \begin{cases} r = 2 \\ \theta = \frac{\pi}{4} + \frac{k\pi}{2} \end{cases}, \boxed{k = 0,1,2,3}$$

$$z_0 = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 2\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = \underline{\sqrt{2} + i\sqrt{2}}$$

$$z_1 = 2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = 2\left(-\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = \underline{-\sqrt{2} + i\sqrt{2}}$$

$$z_2 = 2\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) = 2\left(-\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) = \underline{-\sqrt{2} - i\sqrt{2}}$$

$$z_3 = 2\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = 2\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) = \underline{\sqrt{2} - i\sqrt{2}}.$$



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(17) $$\left(\frac{x^4}{a} - \frac{1}{x}\right)^9 = \sum_{k=0}^{9} \underbrace{\binom{9}{k}\left(\frac{x^4}{a}\right)^k\left(\frac{-1}{x}\right)^{9-k}}_{\alpha_k}$$

$$\alpha_k = \binom{9}{k} \cdot \frac{x^{4k}}{a^k} \cdot \frac{(-1)^{9-k}}{x^{9-k}} = \binom{9}{k} x^{\boxed{5k-9}} \cdot \frac{(-1)^{9-k}}{a^k}$$

Vi vill ha en term $-4x$ av grad 1. Då

måste $5k-9=1$ dvs. $\underline{k=2}$.

Men $$\alpha_2 = \binom{9}{2}\frac{(-1)^7}{a^2}x = \frac{9\cdot 8}{2}\cdot\frac{(-1)}{a^2}x = -\frac{36}{a^2}x$$

och detta är $-4x$ om $\underline{a=\pm 3}$ Svar: $\boxed{a=\pm 3}$

(18.) $$\lg\frac{100}{x+10} - \lg(x-5) = 0 \iff \lg\frac{100}{(x+10)(x-5)} = 0$$

$$\iff \frac{100}{(x+10)(x-5)} = 1 \iff (x+10)(x-5) = 100 \iff$$

$$x^2-5x+10x-50 = 100 \iff x^2+5x-150 = 0 \iff$$

$$x = -\frac{5}{2}\pm\sqrt{\frac{25}{4}+150} = -\frac{5}{2}\pm\sqrt{\frac{625}{4}} = -\frac{5}{2}\pm\frac{25}{2} = \begin{cases}-15\\+10\end{cases}$$

Prövning: Ingen av termerna är definierad (reellt) för $x=-15$, men båda fungerar för $x=10$.

Svar: $\boxed{x=10}$
