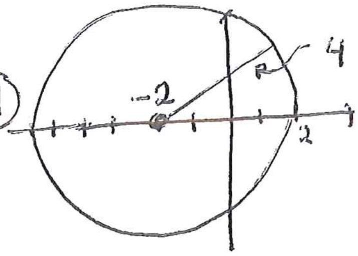
SVAR!

BASKURSEN
TENTA
18 Augusti 2011

LÖSNINGAR!!

(1) $8x$. (2) 0. (3) 45. (4)



(5) 3. (6) $\sqrt{2}$ (7) 1.

(8) $x \leq -1$ eller $x \geq 3$.

(9) $x^3 - 2x^2 - 3x = 0 \iff x(x^2 - 2x - 3) = 0$ ger $x = 0$ eller

$x^2 - 2x - 3 = 0 \iff x = 1 \pm \sqrt{1+3} = 1 \pm 2$ $\boxed{\text{Svar: } x = 0, -1, 3}$

(10) Basfall: $n = 0$.

$VL = \sum_{k=0}^{0} 2^k = 2^0 = 1$, $HL = 2^{0+1} - 1 = 2 - 1 = 1$ OK.

Utgå ifrån: $\boxed{\sum_{k=0}^{n} 2^k} = 2^{n+1} - 1$ (A) och visa att då följer

(B) $\sum_{k=0}^{n+1} 2^k = 2^{(n+1)+1} - 1$. $VL(B) = \sum_{k=0}^{n+1} 2^k = \boxed{\sum_{k=0}^{n} 2^k} + 2^{n+1} =$ (enligt A) $=$

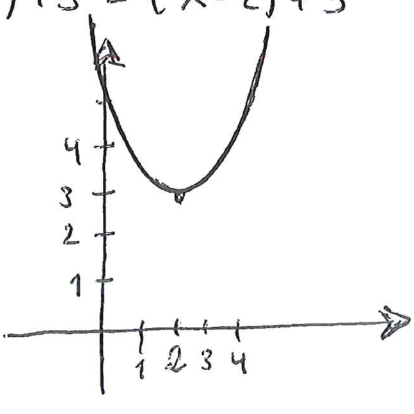
$= (2^{n+1} - 1) + 2^{n+1} = 2 \cdot 2^{n+1} - 1 = 2^{n+2} - 1 = HL(B)$!

Induktionsaxiomet ger resultatet. □.

(11) $$\frac{(2+i)(2-i)}{(1+i)} = \frac{5}{1+i} = \frac{5(1-i)}{(1+i)(1-i)} = \frac{5-5i}{2} = \boxed{\frac{5}{2} - \frac{5i}{2}}$$

(12) $y = x^2 - 4x + 7 = (x^2 - 4x + 4) + 3 = (x-2)^2 + 3$

Vertex: $\boxed{(2,3)}$.



(13) $\dfrac{(3+\sqrt{5})(3-\sqrt{5})}{\sqrt{75}-\sqrt{3}} = \dfrac{3^2-5}{\sqrt{3\cdot 25}-\sqrt{3}} = \dfrac{4}{5\sqrt{3}-\sqrt{3}} = \dfrac{4}{4\sqrt{3}} = \boxed{\dfrac{1}{\sqrt{3}}}$.

(14) Logaritmera VL: $\underline{\lg\left(a^{\lg b}\right) = \lg b \cdot \lg a} = \lg a \cdot \lg b = \lg\underline{\left(b^{\lg a}\right)}$!

(15) Binomialsatsen ger:

$\left(\frac{1}{x}-x\right)^5 = \left(\frac{1}{x}+(-x)\right)^5 = \sum_{k=0}^{5}\binom{5}{k}\left(\frac{1}{x}\right)^k(-x)^{5-k} =$

$\sum_{k=0}^{5}\binom{5}{k}x^{-k}\cdot x^{5-k}\cdot(-1)^{5-k} = \sum_{k=0}^{5}\binom{5}{k}(-1)^{5-k}x^{\boxed{5-2k}}$.

Förstagradstermen svarar mot $k=2$ $(5-2\cdot 2=1)$,

så termen blir $\binom{5}{2}(-1)^3\cdot x = -10x$ Svar: $\boxed{-10x}$

(16) $\cos^2 2x + 4\sin^2 x = 1 \iff$ (Dubbla vinkeln!) $\iff$

$(1-2\sin^2 x)^2 + 4\sin^2 x = 1 \iff 1 - 4\sin^2 x + 4\sin^4 x + 4\sin^2 x = 1$

$\iff 4\sin^4 x = 0 \iff \sin x = 0 \iff \boxed{x = n\cdot\pi,\ n\in\mathbb{Z}}$

(17) $\underline{z-1=w}$ ger $w^3 = 8 \iff \begin{cases} w = r(\cos\theta + i\sin\theta) \\ w^3 = r^3(\cos 3\theta + i\sin 3\theta) \end{cases}$ $\Big|$ $8 = 8(\cos 0 + i\sin 0)$

så ekvationen blir:

$r^3(\cos 3\theta + i\sin 3\theta) = 8(\cos 0 + i\sin 0) \iff \begin{cases} r^3 = 8 \\ 3\theta = 0 + 2n\pi \end{cases} \Rightarrow \begin{cases} r = 2 \\ \theta = \dfrac{2n\pi}{3} \end{cases}$

Så: $\ _k = 2\left(\cos\frac{2k\pi}{3} + i\sin\frac{2k\pi}{3}\right)$

$\begin{cases} z_0 = 1 + 2(\cos 0 + i\sin 0) = 3 \\ z_1 = 1 + 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 1 + 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = i\sqrt{3} \\ z_2 = 1 + 2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) = 1 + 2\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = -i\sqrt{3} \end{cases}$

Svar: Rötter. $3,\ \pm i\sqrt{3}$

(18) Ex: $z(z-(1-i))(z-(1+i)) = z(z^2-2z+2) = \boxed{z^3-2z^2+2z}$

— x —