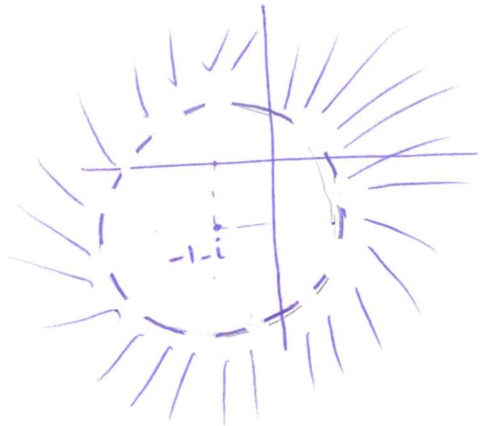
Facit:
tenta 2012-08-28.

(1) $-\sqrt{2}/2$ (2). $\frac{98}{9}$ (3). $\frac{4x}{4x^2-1}$

(4). $|z-(-1-i)| > 2$.



(5) $\begin{cases} x = 0 + k\pi, & k \in \mathbb{Z} \\ x = \pi/2 + k\pi, & k \in \mathbb{Z} \end{cases}$

(6) $(x-2)^2 + y^2 = 9 \longrightarrow$ radie $= 3$, medelpunk $= (2,0)$.

(7). $\frac{10}{11}$ (8) $x = 0$, $x = 1$.

(9). om $x \geq 0$: $x - 2 \leq x \rightarrow -2 \leq 0$ det stämmer altid.

svar för den delen är $x \geq 0$.

om $x < 0$: $-x - 2 \leq x \rightarrow 2x \geq -2 \rightarrow x \geq -1$

svar för den delen $-1 \leq x < 0$

svar: $x \geq 0$ eller $-1 \leq x < 0$ så svar blir $\boxed{x \geq -1}$

(10) basfall: $n=1$

$$\left.\begin{array}{l} VL = 6.1+2=8 \\ HL = 3.1^2+5.1=8 \end{array}\right\} \Rightarrow VL = HL \quad \text{ok!}$$

Visa: $\sum_{K=1}^{P} (6K+2) = 3p^2+5p \Rightarrow \sum_{K=1}^{P+1} (6K+2) = 3(P+1)^2+5(P+1).$

(A) (B)

$$VL(B) = \sum_{K=1}^{P+1}(6K+2) = \sum_{K=1}^{P}(6k+2) + 6(P+1)+2 = (A) =$$

$$3p^2+5p+6(P+1)+2 = 3p^2+6p+3+5p+5$$

$$= 3(p^2+2p+1)+5(p+1) = 3(p+1)^2+5(p+1) = HL(B).$$

Induktionssteget visat, och formeln gäller för $n \geq 1$ enligt induktion axiomet.

(11) $\dfrac{6!}{3!\,2!} = 60..$

(12) $-1+i = \sqrt{2}\left(\cos(3\pi/4)+i\sin(3\pi/4)\right).$

$1+i\sqrt{3} = 2\left(\cos(\pi/3)+i\sin(\pi/3)\right)$

$\sqrt{3}-i = 2\left(\cos(-\pi/6)+i\sin(-\pi/6)\right).$

$$\frac{(-1+i)(1+i\sqrt{3})}{\sqrt{3}-i} = \sqrt{2}\left(\cos(5\pi/4)+i\sin(5\pi/4)\right).$$

(13) $(x-3)^2+8y=0$

sätter $y^2=x-3$: $(y^2)^2+8y=0 \rightarrow y^4+8y=0$

$y=0$, $y=-2$

$\downarrow$ $\downarrow$

$x=3$ $x=7$

$\downarrow$ $\downarrow$

$(3,0)$ $(7,-2)$.

(14) $3^{2x}-1=80 \rightarrow 3^{2x}=81 \rightarrow 2x=\log_3 81=4 \rightarrow x=2$

(15) $2\sin x\cos x+\sin x=0 \rightarrow \sin x(2\cos x+1)=0$

$\sin x=0$ $\qquad$ $2\cos x+1=0$

$x=k\pi$ $\qquad$ $\cos x=-1/2$

$$\begin{cases} x=\pi-\pi/3+2k\pi \\ x=\pi+\pi/3+2k\pi. \end{cases}$$

(16) $z^5=r^5(\cos 5\theta+i\sin 5\theta)$

$32=2^5(\cos 0+i\sin 0)$.

$r=2$, $\quad 5\theta=0+2k\pi \rightarrow \theta=\frac{2}{5}k\pi$

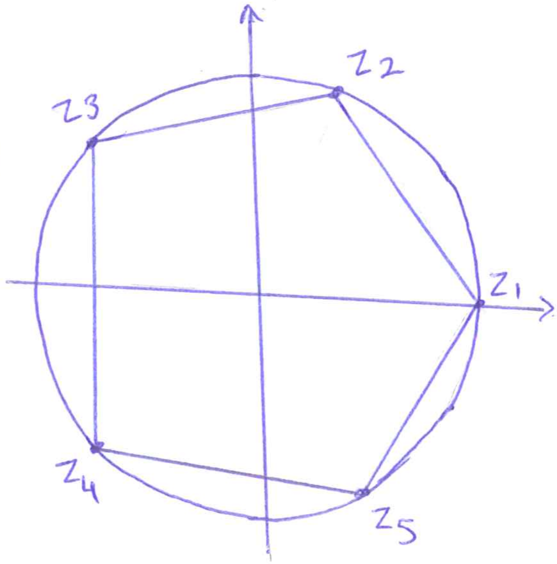
$k=0 \rightarrow z_1=2(\cos 0+i\sin 0)=2$

$k=1 \rightarrow z_2=2(\cos 2\pi/5+i\sin 2\pi/5)$

$k=2 \rightarrow z_3=2(\cos 4\pi/5+i\sin 4\pi/5)$

$K=3 \rightarrow z_4 = 2\left(\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}\right)$

$K=4 \rightarrow z_5 = 2\left(\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}\right)$



(17) $\frac{2x+4}{x-1} - x > 0 \rightarrow \frac{-x^2+3x+4}{x-1} > 0 \rightarrow \frac{-(x+1)(x-4)}{(x-1)} > 0$

$\rightarrow \frac{(x+1)(x-4)}{x-1} < 0$

x		-1		1		4	
$x+1$	$- - -$	0	$+ +$	$+ +$	$+$		$+ +$
$x-1$	$- - -$		$- -$	0	$+ +$		$+ +$
$x-4$	$- -$	$- -$	$- -$	$- -$	$- -$	0	$+ +$
$\frac{(x+1)(x-4)}{x-1}$	$-$	0	$+$	$*$	$-$	0	$+$

Svar: $x < -1$ eller $1 < x < 4$.

(18) $P(z) = z^4 - 2z^3 + z^2 + 6z - 12$

$P(1+i\sqrt{3}) = 0$ $\qquad$ $P(1-i\sqrt{3}) = 0$

$$\left(z-(1+i\sqrt{3})\right)\left(z-(1-i\sqrt{3})\right) = z^2 - 2z + 4$$

$$\begin{array}{l|l}
\quad z^2 - 3 & \\
\hline
z^4 - 2z^3 + z^2 + 6z - 12 & z^2 - 2z + 4 \\
-(z^4 - 2z^3 + 4z^2) & \\
\hline
\quad -3z^2 + 6z - 12 & \\
\quad -(-3z^2 + 6z - 12) & \\
\hline
\quad 0 &
\end{array}$$

$z^2 - 3 = 0 \longrightarrow z = \pm\sqrt{3}$.