2012-10-26

BASKURS

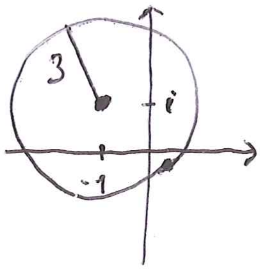
A

1. $\boxed{1/\sqrt{2}}$

2. $\boxed{\frac{3}{2}(x-2)}$

3. $\boxed{15}$

4.



5. $\boxed{2}$

6. $\boxed{x=\frac{\pi}{2}+n\pi}$

7. $\boxed{-4<x<2}$

8. $\boxed{\text{Punkten } (-3,24)}$

B

9. Basfall: $VL(n=0)=\sum_{k=0}^{0}(1-2^k)=1-2^0=1-1=0$

$HL(n=0)=0+2(1-2^0)=0+2(1-1)=0$

Induktionssteg. Visa

$$\underbrace{\sum_{k=0}^{n_0}(1-2^k)=n_0+2(1-2^{n_0})}_{A} \Rightarrow \underbrace{\sum_{k=0}^{n_0+1}(1-2^k)=(n_0+1)+2(1-2^{n_0+1})}_{B}$$

$$\boxed{VL(B)}=\sum_{k=0}^{n_0+1}(1-2^k)=\sum_{k=0}^{n_0}(1-2^k)+(1-2^{n_0+1})\overset{\boxed{\text{ur } A}}{=}n_0+2(1-2^{n_0})+(1-2^{n_0+1})$$

$$=(n_0+1)+2(1-2^{n_0})-2^{n_0+1}=(n_0+1)+2\cdot1-2\cdot2^{n_0}-2^{n_0+1}=$$

$$=(n_0+1)+2\cdot1-2^{n_0+1}-2^{n_0+1}=(n_0+1)+2\cdot1-2\cdot2^{n_0+1}=$$

$$=(n_0+1)+2(1-2^{n_0+1})=\boxed{HL(B)}$$

Så induktionssteget visat och resultatet följer ur induktionsaxiomet.

10. $\frac{2-x}{1+2x} \geq -1 \iff \frac{2-x}{1+2x} + 1 \geq 0 \iff \frac{2-x}{1+2x} + \frac{1+2x}{1+2x} \geq 0 \iff$

$\frac{2-x+1+2x}{1+2x} \geq 0 \iff \frac{3+x}{2(x+\frac{1}{2})} \geq 0.$

x	-3	$-\frac{1}{2}$
$3+x$	$- - 0 + + + + + + + +$	
$2(x+\frac{1}{2})$	$- - - - - 0 + + + + + +$	
$\frac{3+x}{2(x+\frac{1}{2})}$	$+ + + 0 - - * + + + + + +$	

Svar: Olikheten är uppfylld om $\underline{x \leq -3}$ eller $\underline{x > -\frac{1}{2}}$.

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11. $\frac{2i}{1+i} = \frac{2i(1-i)}{(1+i)(1-i)} = \frac{2i(1-i)}{2} = i(1-i) = 1+i$

$|1+i| = \sqrt{1^2+1^2} = \sqrt{2}$ så $1+i = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = \boxed{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}$

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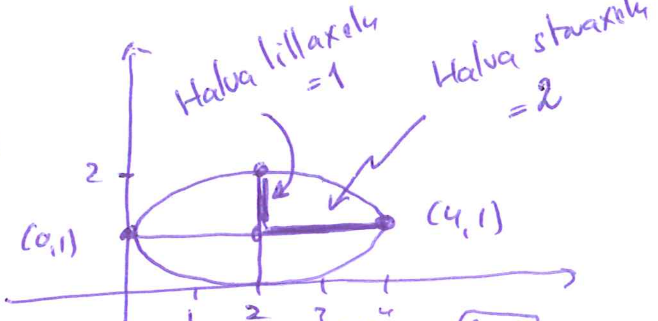
12. Ur informationen till höger ser vi att ekvationen blir:

$$\boxed{\frac{(x-2)^2}{2^2} + \frac{(y-1)^2}{1^2} = 1}$$



Mittpunkt är $\boxed{(2,1)}$

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13. $x^3 + 4x^2 = 7x \iff x^3 + 4x^2 - 7x = 0 \iff$

$x(x^2+4x-7) = 0$ ger $\boxed{x=0}$ och $x^2+4x-7=0 \iff$

$x = -2 \pm \sqrt{4+7} = \underline{-2 \pm \sqrt{11}}$

Svar: Rötter: $x=0$, $x = -2 \pm \sqrt{11}$

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14. $\log_2 \frac{x}{2x+1} - \log_2 \frac{1}{x+3} = 1 \iff \left(\log \log 2\right) \iff$

$$\log_2 \frac{\frac{x}{2x+1}}{\frac{1}{x+3}} = 1 \iff \frac{x(x+3)}{2x+1} = 2 \iff$$

$$x(x+3) = 2(2x+1) \iff x^2+3x = 4x+2 \iff x^2-x-2=0$$

$$\iff x = \frac{1}{2} \pm \sqrt{\frac{1}{4}+2} = \frac{1}{2} \pm \sqrt{\frac{9}{4}} = \frac{1}{2} \pm \frac{3}{2} = \begin{cases} 2 \\ -1 \end{cases}$$

OBS! Båda uttrycken i den ursprungliga ekvationen är definierade för både $x=2$ och $x=-1$ - logaritmen är bara definierad för positiva tal.

C. $8z^3 = i \iff z^3 = \frac{1}{8} i$. Polär form

(15) $z = r(\cos\theta + i\sin\theta) \Rightarrow z^3 = r^3(\cos 3\theta + i \sin\theta)$ (Moivres Formel)

$\frac{1}{8} i = \frac{1}{8}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ så ekvationen blir:

$$r^3(\cos 3\theta + i\sin 3\theta) = \frac{1}{8}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \Rightarrow$$

$$\begin{cases} r^3 = 1/8 \\ 3\theta = \frac{\pi}{2} + 2k\pi \end{cases} \qquad \begin{aligned} r &= 1/2 \\ \theta &= \pi/6 + \frac{2k\pi}{3} \end{aligned}$$

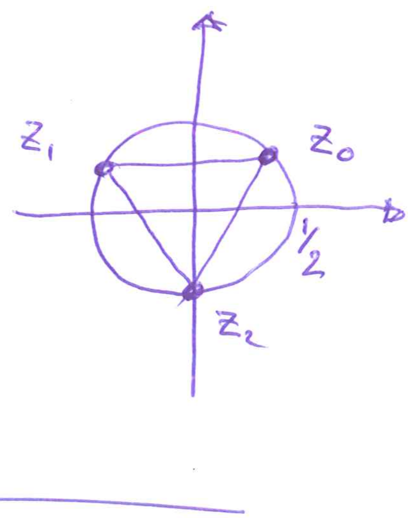
$k = 0, 1, 2$

$$\boxed{z_k = \frac{1}{2}\left(\cos\left(\frac{\pi}{6} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{\pi}{6} + \frac{2k\pi}{3}\right)\right)}$$

$$z_0 = \frac{1}{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \frac{1}{2}\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = \boxed{\frac{\sqrt{3}}{4} + \frac{i}{4}}$$

$$z_1 = \frac{1}{2}\left(\cos\left(\underbrace{\frac{\pi}{6} + \frac{2\pi}{3}}_{\frac{5\pi}{6}}\right) + i\sin\left(\frac{\pi}{6} + \frac{2\pi}{3}\right)\right) = \boxed{-\frac{\sqrt{3}}{4} + \frac{i}{4}}$$

$$z_2 = \frac{1}{2}\cos\left(\underbrace{\frac{\pi}{6} + \frac{4\pi}{3}}_{\frac{3\pi}{2}}\right) + i\sin\left(\frac{\pi}{6} + \frac{4\pi}{3}\right)) = \boxed{-\frac{1}{2} i}$$



16.

$\sin^2 x - \cos^2 x = \sin 2x \Leftrightarrow -(\cos^2 x - \sin^2 x) = \sin 2x$

$\Leftrightarrow$ (Dubbla vinkeln!) $-\cos 2x = \sin 2x \Leftrightarrow$

$\cos 2x = -\sin 2x \Leftrightarrow \cos 2x = \sin(-2x)$ [sinus udda!]

$\Leftrightarrow$ [använd t.ex. $\sin\alpha = \cos(\pi/2 - \alpha)$]

$\cos 2x = \cos\left(\frac{\pi}{2} - (-2x)\right) \Leftrightarrow \underline{\cos 2x = \cos\left(\frac{\pi}{2} + 2x\right)}$

Fall a) $2x = \frac{\pi}{2} + 2x + 2k\pi \Leftrightarrow 0 = \frac{\pi}{2} + 2k\pi$ OMÖJLIGT.

b) $2x = -\left(\frac{\pi}{2} + 2x\right) + 2k\pi \Leftrightarrow 4x = -\frac{\pi}{2} + 2k\pi$

$\Leftrightarrow \boxed{x = -\pi/8 + k\pi/2}$

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17. $\left(x^2 + \frac{2}{x}\right)^{11} = \sum_{k=0}^{11} \binom{11}{k} \underbrace{(x^2)^k \left(\frac{2}{x}\right)^{11-k}}_{\alpha_k}$ Binomialsatsen!

Vad skall k vara för att x skall ha potens 1?

$\alpha_k = x^{2k} \cdot \frac{2^{11-k}}{x^{11-k}} =$

$= x^{2k-(11-k)} \cdot 2^{11-k} = x^{3k-11} \cdot 2^{11-k}$

Detta blir x precis när $\underline{k=4}$.

så förstagradstermen är den då $\underline{k=4}$:

$\boxed{\binom{11}{4} \cdot 2^7 \cdot x}$

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18. Polynomet har reella koefficienter och

då är även $\overline{z} = 1 - i\sqrt{3}$ en rot och

enl. FAKTORSATSEN är polynomet delbart

med $(z-(1+i\sqrt{3}))(z-(1-i\sqrt{3})) = z^2 - z(1-i\sqrt{3}) - z(1+i\sqrt{3})$

$+ (1+i\sqrt{3})(1-i\sqrt{3}) = \underline{z^2 - 2z + 4}$

Metod 1 (Tänk!)

$$z^4 - 3z^3 - 6z^2 + 20z - 48 = (z^2 - 2z + 4)\underline{(z^2 - z - 12)}$$

Metod 2: "Stolen"

$$\begin{array}{l|l}
\multicolumn{2}{l}{z^2 - z - 12} \\
\hline
z^4 - 3z^3 - 6z^2 + 20z - 48 & z^2 - 2z + 4 \\
-z^4 + 2z^3 - 4z^2 & \\
\hline
-z^3 - 10z^2 + 20z & \\
z^3 - 2z^2 + 4z & \\
\hline
-12z^2 + 24z - 48 & \\
+12z^2 - 24z + 48 & \\
\hline
0 &
\end{array}$$

Lös till sist. $z^2 - z - 12 = 0$ $\quad z = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 12} =$

$= \frac{1}{2} \pm \sqrt{\frac{49}{4}} = \frac{1}{2} \pm \frac{7}{2} = \begin{cases} 4 \\ -3 \end{cases}$

Svar: $\begin{cases} z = 1 \pm i\sqrt{3} \\ z = 4,\ z = -3 \end{cases}$