BASKURS 2013-01-19

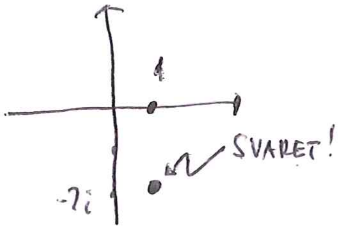
SVAR!

A

1. $\boxed{1/2}$

2. $\boxed{\dfrac{x}{x-4}}$

3. $\boxed{15}$

5. 

6. $\boxed{x = \dfrac{3\pi}{8} + \dfrac{n\pi}{2}}$

7. $\boxed{x = -2, 3}$

8. $\boxed{\text{Radie } 2}$

B

9. Basfall: $\underline{n=0}$ $\begin{cases} VL = \sum_{k=0}^{0} 3\cdot 4^k = 3\cdot 4^0 = 3\cdot 1 = 3 \\ HL = 4^{0+1} - 1 = 4^1 - 1 = 4 - 1 = 3 \end{cases}$ OK!

Visa: $\underbrace{\sum_{k=0}^{n_0} 3\cdot 4^k = 4^{n_0+1} - 1}_{(A)} \Rightarrow \underbrace{\sum_{k=0}^{n_0+1} 3\cdot 4^k = 4^{(n_0+1)+1} - 1}_{(B)}.$

$$VL(B) = \sum_{k=0}^{n_0+1} 3\cdot 4^k = \left(\sum_{k=0}^{n_0} 3\cdot 4^k\right) + 3\cdot 4^{n_0+1} = \left(\begin{matrix}\text{Eftersom (A)} \\ \text{antas sann!}\end{matrix}\right) =$$

$$= \left(4^{n_0+1} - 1\right) + 3\cdot 4^{n_0+1} = 4^{n_0+1} + 3\cdot 4^{n_0+1} - 1 = 4^{n_0+1}(1+3) - 1 =$$

$$= 4\cdot 4^{n_0+1} - 1 = \underline{4^{n_0+2} - 1} = HL(B)$$

Så om påståendet gäller för ett visst tal n_0, så gäller det även för n_0+1. Induktionsaxiomet ger då att formeln gäller för alla $n \in \mathbb{N}$. $\square$

10.

$$\frac{3x-5}{1-2x} \geq 2 \Leftrightarrow \frac{3x-5}{1-2x} - 2 \geq 0 \Leftrightarrow \frac{3x-5-2(1-2x)}{1-2x} \geq 0$$

$$\Leftrightarrow \frac{7x-7}{1-2x} \geq 0 \Leftrightarrow \frac{7(x-1)}{2(\frac{1}{2}-x)} \geq 0 \quad (1)$$

Teckenschema

x		$\frac{1}{2}$		1	
$x-1$	$-$	$-$	$-$	0	$+$
$\frac{1}{2}-x$	$+$	0	$-$	$-$	$-$
$\frac{x-1}{\frac{1}{2}-x}$	$-$	$*$	$+$	0	$-$

Vi ser att (1) är uppfylld om:

Svar: Olikheten gäller om $\boxed{\frac{1}{2} < x \leq 1}$

11.

$$\frac{i(1+i)}{1-i} = \frac{i-1}{1-i} = \frac{(-1)(1-i)}{1-i} = \underline{\underline{-1}}$$ så polärt:

$$-1 = 1\cdot(\cos\pi + i\sin\pi) \left(= \underline{\underline{e^{\pi i}}}\right).$$

Svar: $\boxed{1(\cos\pi + i\sin\pi)}$

12.

$$\ln(VL) = \ln(a^{\ln b}) = \ln b \cdot \ln a = \ln a \cdot \ln b = \ln(b^{\ln a}) = \ln(HL)$$

så $\underline{VL = HL}$!

13.

5^{3} ← antal val.

↑ antal valmöjligheten

dvs. $\underline{125}$.

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14. $5^{2x}+1=2\cdot 5^x \Leftrightarrow (5^x)^2+1=2\cdot 5^x$.

Sätt $5^x=t$: $t^2+1=2t \Rightarrow t^2-2t+1=0$

$\Leftrightarrow (t-1)^2=0$, $\underline{t=1}$ (dubbelrot)

$5^x=t$ ger $5^x=1 \Rightarrow \boxed{x=0}$

$\boxed{C}$ 15. $z^4=-16$. Polär form: $z=r(\cos\theta+i\sin\theta) \Rightarrow$

$$\begin{cases} z^4=r^4(\cos 4\theta+i\sin 4\theta) & \text{(Moivres formel)} \\ -16=16(\cos\pi+i\sin\pi) \end{cases}$$

Så vi får $\begin{cases} r^4=16 \\ 4\theta=\pi+2k\pi \end{cases}$ $\begin{cases} r=2 \\ \theta=\frac{\pi}{4}+\frac{k\pi}{2} \end{cases}$ $k=0,1,2,3$

Så lösningarna ges av

$$z_k=2\left(\cos\left(\frac{\pi}{4}+\frac{k\pi}{2}\right)+i\sin\left(\frac{\pi}{4}+\frac{k\pi}{2}\right)\right) \quad k=0,1,2,3.$$

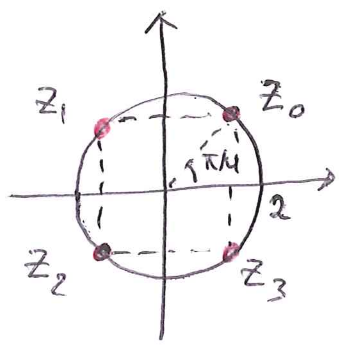
eller

$z_0=\sqrt{2}+i\sqrt{2}$

$z_1=-\sqrt{2}+i\sqrt{2}$

$z_2=-\sqrt{2}+i(-\sqrt{2})$

$z_3=\sqrt{2}-i\sqrt{2}$



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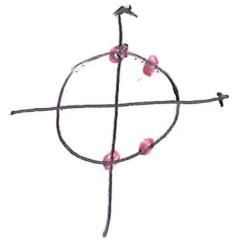
16. Dubbla vinkeln : $\cos 2x = 2\cos^2 x - 1$ ger

$$2\cos^2 x - 1 = \cos x - 1 \iff 2\cos^2 x - \cos x = 0 \iff$$

$$\cos x\,(2\cos x - 1) = 0 \iff$$

$$\begin{cases} \cos x = 0 \Rightarrow x = \frac{\pi}{2} + n\pi \\ \cos x = \frac{1}{2} \Rightarrow x = \pm\frac{\pi}{3} + 2n\pi \end{cases}$$



Svar: $\begin{cases} x = \pi/2 + n\pi \\ x = \pm \pi/3 + 2n\pi \end{cases}$

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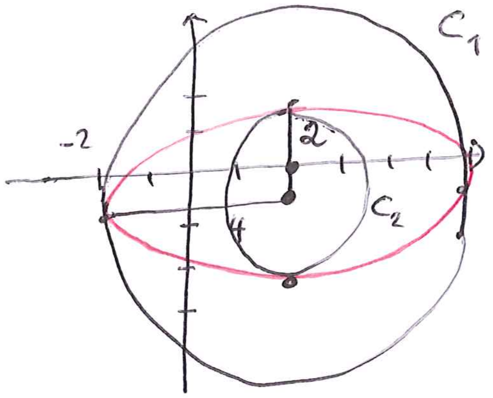
17. $$(x^2 - 4x + 4) - 4 + 4(y^2 + y + 1/4) - 1 = 11$$

KVADRATKOMPLETTERA!

$$(x-2)^2 + 4(y+1/2)^2 = 16 \iff \frac{(x-2)^2}{4^2} + \frac{(y+1/2)^2}{2^2} = 1$$

$\begin{cases} \text{Halva storaxeln} = 4 \\ \text{Halva lillaxeln} = 2 \end{cases}$



Omskrivna cirkeln C_1 har area $\pi \cdot 4^2 = 16\pi$

Inskrivna cirkeln C_2 har area $\pi \cdot 2^2 = 4\pi$

$$\text{Area}(C_1)/\text{Area } C_2 = \frac{16\pi}{4\pi} = \boxed{4}$$

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18. $x^3 - ax = 6$. $x = -2$ rot.

Sätt in $x = -2$:

$(-2)^3 - a(-2) = 6 \Leftrightarrow -8 + 2a = 6 \Leftrightarrow \underline{\underline{a = 7}}$.

Ekvationen blir:

$x^3 - 7x - 6 = 0$ och $x = -2$ är

en rot. Faktorsatsen ger att

$x^3 - 7x - 6$ är delbart med $x - (-2) = x + 2$.

Alt. 1: $x^3 - 7x - 6 = (x+2)\underbrace{(x^2 - 2x - 3)}$

$x = 1 \pm \sqrt{1+3} = 1 \pm 2 = \begin{cases} 3 \\ -1 \end{cases}$

$\boxed{\text{Svar: } a = 7, \text{ rötter: } -2, -1, 3}$

Al. 2.

$$
\begin{array}{l|l}
\multicolumn{1}{l}{x^2 - 2x - 3} & \\
\hline
x^3 - 7x - 6 & x + 2 \\
-x^3 - 2x^2 & \\
\hline
-2x^2 - 7x & \\
+2x^2 + 4x & \\
\hline
-3x - 6 & \\
3x + 6 & \\
\hline
0 &
\end{array}
$$