BASKURSEN 25 okt. 2013

Svar o Lösningar

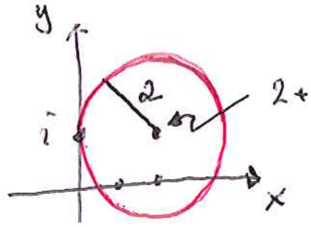
A.

1. $\boxed{1/\sqrt{2}}$

2. $\boxed{\dfrac{3}{3-x}}$

3. $\boxed{\dfrac{15}{8}}$

4.



5. $\boxed{2}$

6. $\boxed{56}$

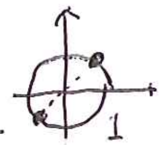
7. $\boxed{\{x;\ -7 \le x \le 3\}}$

8. $\boxed{2\sqrt{3}}$ $(=\sqrt{12})$

B

9. $2\sin x \cdot \cos x = 1 \Leftrightarrow$ (Dubbla vinkeln) $\Leftrightarrow \sin 2x = 1 \Leftrightarrow$

$$2x = \frac{\pi}{2} + 2n\pi \Leftrightarrow \boxed{x = \frac{\pi}{4} + n\pi,\ n \in \mathbb{Z}}$$



10. $$\frac{2x-1}{3-x} \le 3 \Leftrightarrow \frac{2x-1}{3-x} - 3 \le 0 \Leftrightarrow \frac{2x-1}{3-x} - \frac{3(3-x)}{3-x} \le 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{2x-1-3(3-x)}{3-x} \le 0 \Leftrightarrow \frac{2x-1-9+3x}{3-x} \le 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{5x-10}{3-x} \le 0 \Leftrightarrow \frac{5(x-2)}{3-x} \le 0 \quad (*)$$

Teckenschema

x		2		3	
$x-2$	$- - -$	0	$+ +$	$+$	$+$
$3-x$	$+ + +$	$+$	$+$	0	$- -$
$\frac{5(x-2)}{3-x}$	$- - -$	0	$+ +$	$*$	$- - -$

SLUTSATS: Olikheten gäller om

$$\boxed{x \le 2 \text{ eller } x > 3}$$

11. $\sqrt{6} + i\sqrt{2}$ har belopp $\sqrt{(\sqrt{6})^2 + (\sqrt{2})^2} = \sqrt{6+2} = \sqrt{8} = 2\sqrt{2}$ så

$$\sqrt{6} + i\sqrt{2} \overset{\text{BUB}}{=} 2\sqrt{2}\left(\frac{\sqrt{6}}{2\sqrt{2}} + i\frac{\sqrt{2}}{2\sqrt{2}}\right) = 2\sqrt{2}\left(\frac{\sqrt{3}\cdot\sqrt{2}}{2\sqrt{2}} + i\frac{\sqrt{2}}{2\sqrt{2}}\right) = 2\sqrt{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) =$$

$$= \boxed{2\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)}$$

12. Ekvationen kan skrivas

$$2x^2+3y^2=18 \Leftrightarrow \frac{2x^2}{18}+\frac{3y^2}{18}=1 \Leftrightarrow \frac{x^2}{9}+\frac{y^2}{6}=1 \Leftrightarrow$$

$\dfrac{x^2}{3^2}+\dfrac{y^2}{(\sqrt{6})^2}=1$ och vi ser då att halva storaxeln =
och halva lillaxeln $=\sqrt{6}$ så:

Svar: $\boxed{\begin{array}{lll} \text{Storaxel} & 6 & \text{längdenheter} \\ \text{Lillaxel} & 2\sqrt{6} & \text{"} \end{array}}$

13.

$$x^3-4x=x+2 \Leftrightarrow x^3-4x-(x+2)=0 \Leftrightarrow x(x^2-4)-(x+2)=0$$

$$\Leftrightarrow (\text{K.R.}) \quad x(x-2)(x+2)-(x+2)=0 \Leftrightarrow (x+2)(x(x-2)-1)=0$$

$$\Leftrightarrow (x+2)\cdot(x^2-2x-1)=0$$

Vi löser $x^2-2x-1=0 \Leftrightarrow x=1\pm\sqrt{1+1}=1\pm\sqrt{2}$

Svar: $\boxed{x=-2,\ x=1\pm\sqrt{2}}$

14.

$$\log_4(x+4)-\log_4(x-1)=2 \overset{(\text{LOGLAG 2})}{\Leftrightarrow} \log_4\frac{x+4}{x-1}=2$$

$$\Leftrightarrow \begin{pmatrix}\text{Def. av} \\ \log_4\end{pmatrix} \Leftrightarrow \frac{x+4}{x-1}=4^2 \Leftrightarrow \frac{x+4}{x-1}=16 \Leftrightarrow$$

$$x+4=16(x-1) \Leftrightarrow x+4=16x-16 \Leftrightarrow 20=15x \Leftrightarrow x=20/15=4/3$$

Kontroll: $\log_4(x+4)$ och $\log_4(x-1)$ är båda definierade för $x=4/3$ eftersom $\left(\frac{4}{3}+4\right)$ och $\left(\frac{4}{3}-1\right)$ är positiva.

Svar: $\boxed{x=4/3}$

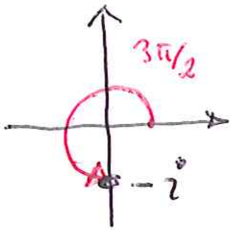
3.

$\boxed{C}$ $z^3 + i = 0 \iff z^3 = -i.$

15. Gå över till polär form:

$z = r(\cos\theta + i\sin\theta) \Rightarrow z^3 = r^3(\cos 3\theta + i\sin 3\theta)$ (Moivre!)

$-i = 1\cdot(0 + i(-1)) = 1\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$



Så ekvationen är:

$$r^3(\cos 3\theta + i\sin 3\theta) = 1\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) \iff$$

$$\begin{cases} r^3 = 1 \\ 3\theta = \frac{3\pi}{2} + 2n\pi \end{cases} \iff \begin{cases} r = 1 \\ \theta = \frac{\pi}{2} + \frac{2n\pi}{3} \quad n = 0, 1, 2 \end{cases}$$

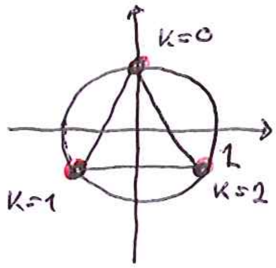
Så lösningarna blir

$$z_k = 1\left(\cos\left(\frac{\pi}{2} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{\pi}{2} + \frac{2k\pi}{3}\right)\right), \quad k = 0, 1, 2$$

$z_0 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = \boxed{i}$

$z_1 = \cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right) = \boxed{-\frac{\sqrt{3}}{2} - \frac{i}{2}}$

$z_2 = \cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right) = \boxed{\frac{\sqrt{3}}{2} - \frac{i}{2}}$



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16. Binomialutveckla:

$$\left(x^2 - \frac{2}{x}\right)^{10} = \sum_{k=0}^{10} \binom{10}{k} \underbrace{(x^2)^k \left(\frac{-2}{x}\right)^{10-k}}_{\alpha_k}$$

$$\alpha_k = (x^2)^k\left(\frac{-2}{x}\right)^{10-k} = x^{2k}\cdot\frac{(-2)^{10-k}}{x^{10-k}} = x^{2k}\cdot x^{-(10-k)}\cdot(-2)^{10-k} =$$

$$= x^{2k-10+k}\cdot(-2)^{10-k} = x^{3k-10}(-2)^{10-k}.$$ Vi får x^2 om

$3k - 10 = 2$ dvs $\boxed{k = 4}$ och koefficienten är då

$$\binom{10}{4}\cdot(-2)^6 = \boxed{\binom{10}{4}\cdot 2^6} \quad (= 210\cdot 64 = 13.440.)$$

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17. (Usch!) Visa: $\sum_{k=1}^{n} (2^{k-1} - 1) = 2^n - n - 1$, $n \in \mathbb{Z}_+$.

Bas fallet: n = 1.
$$\begin{cases} VL(n=1) = \sum_{k=1}^{1} (2^{k-1} - 1) = 2^{1-1} - 1 = 2^0 - 1 = \underline{\underline{0}} \\ HL(n=1) = 2^1 - 1 - 1 = 2 - 2 = \underline{\underline{0}} \end{cases}$$ ok!

Induktionssteg: VISA:
$$\underbrace{\sum_{k=1}^{n_0} (2^{k-1} - 1) = 2^{n_0} - n_0 - 1}_{A} \Rightarrow \underbrace{\sum_{k=1}^{n_0+1} (2^{k-1} - 1) = 2^{n_0+1} - (n_0+1) - 1}_{B}$$

$$\boxed{VL(B)} = \sum_{k=1}^{n_0+1} (2^{k-1} - 1) = \sum_{k=1}^{n_0} (2^{k-1} - 1) + (2^{(n_0+1)-1} - 1) = (\text{Enl. } A) =$$
$$= (2^{n_0} - n_0 - 1) + (2^{n_0} - 1) = (2^{n_0} + 2^{n_0}) - n_0 - 2 = 2 \cdot 2^{n_0} \underbrace{- n_0 - 2}_{-n_0-1-1 = -(n_0+1)-1} =$$
$2^{n_0}(1+1) = 2^{n_0} \cdot 2$
$$= 2^{n_0+1} - (n_0+1) - 1 = \boxed{HL(B)}$$

Så induktionssteget visat och formeln gäller alltså för $n \geq 1$.

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18. Eftersom polynomet har reella koefficienter gäller att även $\bar{z} = (1+i)$ är rot och faktorsatsen säger att polynomet är delbart med $(z-(1-i))(z-(1+i)) =$

$z^2 - z(1+i) - z(1-i) + (1-i)(1+i) = \underline{z^2 - 2z + 2}$

- Utför divisionen:

- Alt 1 (Tank!)

$$z^4 - 2z^3 + 4z - 4 = (z^2 - 2z + 2)(z^2 - 2)$$

forts.

- Alt. 2 (Liggande stolen)

$$
\begin{array}{l|l}
\multicolumn{2}{l}{z^2-2} \\
\hline
z^4-2z^3+4z-4 & z^2-2z+2 \\
\hline
-z^4+2z^3-2z^2 & \\
\hline
\quad -2z^2+4z-4 & \\
\quad +2z^2-4z+4 & \\
\hline
\qquad\qquad 0 &
\end{array}
$$

Polynomet sönderfaller alltså i

$$(z^2-2z+2)(z^2-2) \quad \text{och} \quad z^2-2=0 \Leftrightarrow z=\pm\sqrt{2}$$

Så rötterna är:

$$\boxed{z_{1,2}=1\pm i, \quad z_{3,4}=\pm\sqrt{2}}$$

(Kontroll:
rötternas summa $=2$ OK
rötternas produkt $=-4$ OK!)