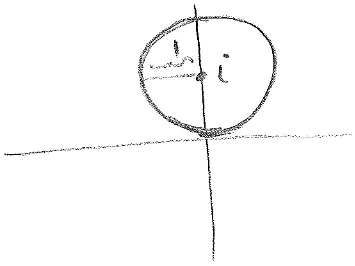
facit tenta 2014-01-10

(1) $\cos \pi/6 = \sqrt{3}/2$.

(2) $\dfrac{x}{x^2-4} - \dfrac{2}{x^2-4} = \dfrac{x-2}{x^2-4} = \dfrac{1}{x+2}$.

(3) $\dfrac{2}{2} + \dfrac{4}{3} + \dfrac{6}{4} = \dfrac{12+16+18}{12} = \dfrac{46}{12}$.

(4)



(5) $\log_5 \dfrac{10^2}{4} = \log_5 25 = 2$.

(6) $\dfrac{12 \cdot 11 \cdot \cancel{10!}}{\cancel{10!}\ 2!} = 66$.

(7) $2 < x < 6$

(8) $\cos 2x = 1 \longrightarrow 2x = 2k\pi \longrightarrow x = k\pi\ ,\ k \in \mathbb{Z}$.

(9) $\dfrac{x^2}{4^2} + \dfrac{y^2}{\left(\frac{4}{\sqrt{6}}\right)^2} = 1$

storaxel: $2 \cdot 4 = 8$

lillaxel: $2 \cdot \dfrac{4}{\sqrt{6}} = \dfrac{8}{\sqrt{6}}$.

(10) $\dfrac{3x+2}{6-x} - 5 \leq 0 \longrightarrow \dfrac{8x-28}{6-x} \leq 0$

x		$28/8$		6	
$8x-28$	$- -$	0	$+ +$	$+$	$+ +$
$6-x$	$+ + +$	$+$	$+$	0	$- - -$
$\dfrac{8x-28}{6-x}$	$- -$	0	$+$	$*$	$- -$

$\boxed{x \leq \frac{28}{8} \text{ eller } x > 6}$

(11) $|3+i\sqrt{3}| = \sqrt{12} = 2\sqrt{3}$

$3+i\sqrt{3} = 2\sqrt{3}\left(\frac{3}{2\sqrt{3}} + i\,\frac{\sqrt{3}}{2\sqrt{3}}\right) = 2\sqrt{3}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= 2\sqrt{3}\left(\cos \pi/6 + i \sin \pi/6\right)$.

(12) $(x-4)^2 + (y+2)^2 = 25$.

$(4-4)^2 + (4+2)^2 = 36 > 25 \rightarrow$ (4,4) ligger utanför cirkeln.

(13) $x(x+2)^2 = x+2 \rightarrow (x(x+2)-1)(x+2) = 0$

$(x^2+2x-1)(x+2) = 0$

$x+2=0$

$x^2+2x-1=0 \rightarrow x = -1 \pm \sqrt{2}$

(14) $\log\frac{(3+x)^4}{16} = 0 \rightarrow \frac{(3+x)^4}{16} = 10^0 = 1$

$(3+x)^4 = 16 \rightarrow 3+x = \pm\sqrt[4]{16} = \pm 2$.

$3+x=2 \rightarrow x=-1$ ok!

$3+x=-2 \rightarrow x=-5$ ej ok!

$\Rightarrow$ $\boxed{x=-1 \text{ är svaret}}$

(15) $z^3 = 8$ $\quad z^3 = r^3(\cos 3\theta + i \sin 3\theta)$

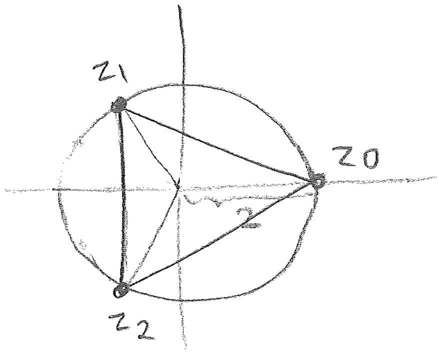
$8 = 8(\cos 0 + i \sin 0)$.

$r=2$, $\theta = \frac{2k\pi}{3}$, $k = 0,1,2$.

$z_0 = 2(\cos 0 + i \sin 0) = 2$

$z_1 = 2(\cos 2\pi/3 + i \sin 2\pi/3) = -1+\sqrt{3}i$

$z_2 = 2\left(\cos \frac{4\pi}{3} + i \sin 4\pi/3\right) = -1-\sqrt{3}i$



(16) -20.

(17) basfallet $VL_1 = 2$, $HL_1 = 2$ ok!

Vi ska visa att

$$\underbrace{\sum_{k=1}^{m} k2^k = (m-1)2^{m+1} + 2}_{A} \Longrightarrow \underbrace{\sum_{k=1}^{m+1} k2^k = m2^{m+2} + 2}_{B}$$

$$VL_B = \sum_{k=1}^{m+1} k2^k = \sum_{k=1}^{m} k2^k + (m+1)2^{m+1} \overset{A}{=} (m-1)2^{m+1} + 2 + (m+1)2^{m+1}$$

$$= 2m \cdot 2^{m+1} + 2 = 2^{m+2} \cdot m + 2 = HL_B.$$

Enligt induktion axiomet formeln gäller för alla naturliga tal $n \geq 1$.

(18) $(z-1+2i)(z-1-2i) = z^2-2z+5$

liggande stol:

$z^4 - 5z^2 + 22z - 30 = (z^2 - 2z+5)(z^2+2z-6)$

$z^2+2z-6 = 0 \rightarrow z = -1 \pm \sqrt{7}$

$z = 1+2i$

$z = 1-2i$