tenta mars 2014

① $\dfrac{2\cancel{(x+3)}}{\cancel{(x+3)}(3-x)} + \dfrac{5}{x-3} = \dfrac{2-5}{3-x} = \dfrac{-3}{3-x} = \dfrac{3}{x-3}$.

② $1-x=3 \to x=-2$, $1-x=-3 \to x=4$.

③ $x=0$, $x^2-2=0 \to x=\pm\sqrt{2}$.

④ $\sum\limits_{k=1}^{3} k^{-2} = 1^{-2} + 2^{-2} + 3^{-2} = 1 + \frac{1}{4} + \frac{1}{9} = \frac{49}{36}$

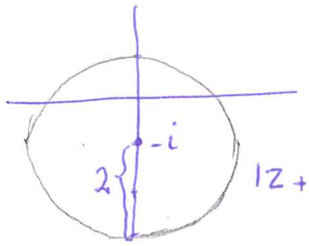
⑤ $y = 2(x^2-4x+4-4)-1 = 2(x-2)^2-9$

vertex : $(2,-9)$.

⑥ $\sin(-7\pi/4) = \frac{1}{\sqrt{2}}$.

⑦ $\log_4 400 - \log_4 100 = \log_4 4 = 1$.

⑧



$|z+i|=2$.

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⑨ $\dfrac{3x-2}{1-x} - 4 \geq 0 \to \dfrac{3x-2-4(1-x)}{1-x} \geq 0$

$\dfrac{7x-6}{1-x} \geq 0$

x		$6/7$		1	
$7x-6$	$----$	0	$+++$	$+$	$++$
$1-x$	$+++$	$+$	$++$	0	$---$
$\frac{7x-6}{1-x}$	$--$	0	$++$	$*$	$--$

$\boxed{\text{Svar: } 6/7 \leq x < 1}$

(10) $\sin x = \frac{1}{\sqrt{2}}$

$$\begin{cases} x = \frac{\pi}{4} + 2k\pi & k \in \mathbb{Z} \\ x = \pi - \pi/4 + 2k\pi \\ \quad = \frac{3\pi}{4} + 2k\pi & k \in \mathbb{Z} \end{cases}$$

$\sin x = \frac{-1}{\sqrt{2}}$

$$\begin{cases} x = -\pi/4 + 2k\pi & k \in \mathbb{Z}. \\ x = \pi + \pi/4 + 2k\pi \\ \quad = \frac{5\pi}{4} + 2k\pi & k \in \mathbb{Z} \end{cases}$$

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(11) $\frac{x^2}{8} + \frac{y^2}{16} = 1 \rightarrow \frac{x^2}{(\sqrt{8})^2} + \frac{y^2}{4^2} = 1$

storaxel : $2.4 = 8$

lillaxeln : $2.\sqrt{8}$.

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(12) $|3 + i\sqrt{3}| = \sqrt{9+3} = \sqrt{12}$

$3 + i\sqrt{3} = \sqrt{12}\left(\frac{3}{\sqrt{12}} + i\frac{\sqrt{3}}{\sqrt{12}}\right) = \sqrt{12}\left(\frac{3}{2\sqrt{3}} + i\frac{\sqrt{3}}{2\sqrt{3}}\right)$

$= \sqrt{12}\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \sqrt{12}\left(\cos \pi/6 + i \sin \pi/6\right)$.

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(13) $\left(\frac{x}{2} - \frac{3}{x^2}\right)^9 = \sum_{k=0}^{9} \binom{9}{k} \left(\frac{x}{2}\right)^{9-k} \left(\frac{-3}{x^2}\right)^k$

$= \sum_{k=0}^{9} \binom{9}{k} \frac{(-3)^k}{2^{9-k}} \cdot x^{9-3k}$

$9 - 3k = 0 \rightarrow 9 = 3k \rightarrow k = 3$

$\binom{9}{3}\frac{(-3)^3}{2^6} = \frac{9.8.7.\cancel{6!}}{3!\ \cancel{6!}} \cdot \frac{-3^3}{2^6} = -\left(\frac{3}{2}\right)^4 \cdot 7$

(14) $\log_3 (2x+1)(x+1) = 1 \rightarrow (2x+1)(x+1) = 3$

$\rightarrow 2x^2 + 3x - 2 = 0 \rightarrow x^2 + \frac{3}{2}x - 1 = 0$

$x = \frac{-3}{4} \pm \sqrt{\frac{9}{16} + 1} = \frac{-3}{4} \pm \frac{\sqrt{25}}{\sqrt{16}} = \frac{-3 \pm 5}{4} \Rightarrow$

$x = -2$ ej ok

$x = \frac{1}{2}$ ok $\Longrightarrow$ svar: $x = \frac{1}{2}$.

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(15) basfallet: $n=0 \rightarrow VL_0 = 3^0 = 1$

$HL_0 = \frac{3^1 - 1}{2} = 1$ ok.

Vi ska visa

$$\underbrace{\sum_{k=0}^{m} 3^k = \frac{3^{m+1} - 1}{2}}_{A} \Longrightarrow \underbrace{\sum_{k=0}^{m+1} 3^k = \frac{3^{m+2} - 1}{2}}_{B}$$

$$VL_B = \sum_{k=0}^{m+1} 3^k = \underbrace{\sum_{k=0}^{m} 3^k}_{VL_A = HL_A} + \sum_{k=m+1}^{m+1} 3^k \overset{A}{=} \frac{3^{m+1} - 1}{2} + 3^{m+1}$$

$$= \frac{3^{m+1} - 1 + 2 \cdot 3^{m+1}}{2} = \frac{3 \cdot 3^{m+1} - 1}{2} = \frac{3^{m+2} - 1}{2} = HL_B$$

Enligt induktion axiomet formeln gäller för alla naturliga tal n.

(16) $z^4 = r^4(\cos 4\theta + i \sin 4\theta)$.

$16i = 16(\cos \pi/2 + i \sin \pi/2)$.

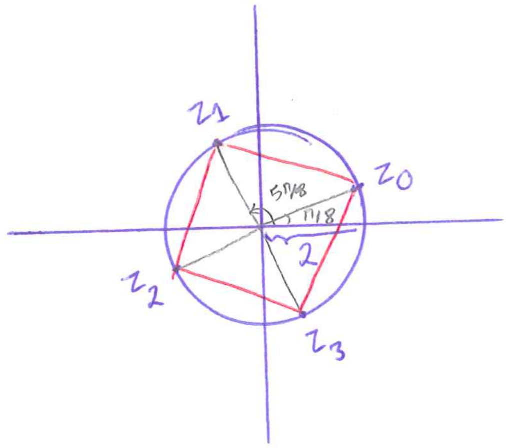
$$\begin{cases} r^4 = 16 \rightarrow r = 2 \\ 4\theta = \pi/2 + 2k\pi \longrightarrow \theta = \pi/8 + \frac{k\pi}{2} \end{cases} \quad , k = 0, 1, 2, 3.$$

$k = 0 \rightarrow z_0 = 2(\cos \pi/8 + i \sin \pi/8)$

$k = 1 \rightarrow z_1 = 2(\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8})$.

$k = 2 \rightarrow z_2 = 2(\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8})$.

$k = 3 \rightarrow z_3 = 2(\cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8})$.



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(17) $y = (x+1)^2 + 3 \rightarrow y - 3 = (x+1)^2$

$(x+1)^2 + (x+1)^4 = 2 \longrightarrow t^2 + t - 2 = 0 \qquad (t = (x+1)^2)$

$t = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + 2} = \frac{-1 \pm 3}{2}$

$t = -2 \longrightarrow (x+1)^2 = -2$ inga reella lösningar.

$t = 1 \longrightarrow (x+1)^2 = 1 \longrightarrow x+1 = \pm 1 \longrightarrow \begin{cases} x = 0 \\ x = -2 \end{cases}$

$x = 0 \rightarrow y = 4 \rightarrow$ $\boxed{(0,4)}$

$x = -2 \rightarrow y = 4 \rightarrow$ $\boxed{(-2,4)}$. skärningspunkterna.

18 $(z-(3i+1))(z-(-3i+1)) = (z-3i-1)(z+3i-1)$

$= z^2 - 2z + 10.$

$$\begin{array}{r|l} z^4 - 2z^3 + 7z^2 + 6z - 30 & z^2 - 2z + 10 \\ \hline -(z^4 - 2z^3 + 10z^2) & z^2 - 3 \\ \hline -3z^2 + 6z - 30 & \\ -3z^2 + 6z - 30 & \\ \hline 0 & \end{array}$$

$z^2 - 3 = 0 \longrightarrow z = \pm\sqrt{3}.$

$$\boxed{\begin{array}{ll} z_1 = 1+3i & z_3 = \sqrt{3} \\ z_2 = 1-3i & z_4 = -\sqrt{3} \end{array}}$$