UPPSALA UNIVERSITY
Department of Mathematics
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Exam in Mathematical Statistics
Inference 1 1MS035
2022–06–14

Time: 8.00-13.00. Limits for the credits 3, 4, 5 are 18, 25 and 32 points, respectively. The solutions should be well motivated.

Permitted aids: Pocket calculator. Dictionary. Formelsamling för stokastik.

1. Let X be a discrete random variable with probability function

$$p_X(x) = \begin{cases} 4\theta^2 & \text{if } x = 1, \\ 4\theta(1-2\theta) & \text{if } x = 2, \\ (1-2\theta)^2 & \text{if } x = 3, \\ 0 & \text{otherwise,} \end{cases}$$

where $0 \leq \theta \leq 1/2$.

We have a random sample $x_1 = 2$, $x_2 = 2$, $x_3 = 3$, $x_4 = 3$ from X.

(a) Find the moment estimate of θ. (1p)

(b) Find the maximum likelihood estimate of θ. (4p)

2. We have a random sample x_1, x_2, x_3 from the random variable X which has expectation $\mu+m$ and variance 1, a random sample y_1, y_2, y_3, y_4 from the random variable Y which has expectation m and variance 1, and one observation z of the random variable Z which has expectation μ and variance 2. The means of the first two samples are denoted by $\bar{x}$ and $\bar{y}$, respectively. We may assume that X, Y, Z are simultaneously independent.

Two estimates of μ are proposed:

$$\mu_1^* = \frac{\bar{x} - \bar{y} + z}{2}, \quad \mu_2^* = 2(\bar{x} - \bar{y}) - z.$$

(a) Show that μ_1^* and μ_2^* are both unbiased for μ. (2p)

(b) Which one of μ_1^* and μ_2^* is most efficient? Motivate your answer. (3p)

3. The number of participants at the Mid summer party in the distant village Gråboda in the heart of the county of Småland in Sweden is assumed to be Poisson distributed with parameter (expectation) λ. The main organizer, Börje (who does not count as a participant) believes that $\lambda = 2$. However, his daughter Börthie (who does not count as a participant either) is more optimistic, and claims that λ must be greater than 2.

(a) It turns out that there are five participants at the party. Is there any evidence that Börthie is right? Try to find out by testing a suitable hypothesis, on the 5% level. (2p)

(b) Calculate the power of the test in (a) in case $\lambda = 6$. (3p)

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4. Eight randomly selected overweight Lagotto dogs are presented with a new diet, to see if this diet makes them lose weight. The table below gives their weights in kilos before and after one week with the diet food.

   Test a suitable hypothesis to try to conclude if the diet works or not. (5p)

Dog no.	1	2	3	4	5	6	7	8
Weight before	20.2	18.3	17.3	21.6	15.1	19.3	19.4	17.6
Weight after	18.3	18.4	14.7	19.8	16.5	17.2	18.2	17.4

5. Zlatan and Tony practice football penalty shoots on the same goal keeper, Hedvig. Among 40 shots each, Zlatan scores on 32 of them, and Tony scores on 24. Say that the probability of scoring on a penalty shot is p_1 for Zlatan and p_2 for Tony.

   (a) Calculate a 95% confidence interval for $p_1 - p_2$. (4p)

   (b) Are Zlatan and Tony equally good at penalty shots? Try to deduce this from the result in (a). What is your conclusion? (1p)

6. The Spanish company El Giant tests the life lengths of an electronic component. Its life length X is assumed to follow an exponential distribution with expectation μ.

   A random sample of 200 components are tested, and their mean life length is 122.3 days.

   (a) Calculate a 99% confidence interval for μ. (3p)

   (b) Calculate a 99% confidence interval for the intensity parameter $\beta = 1/\mu$. (2p)

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7. In the table below, 313 female students are classified according to color of hair (blond or not) and color of eyes (blue or not). Are colors of hair and eyes independent for female students? Perform a suitable hypothesis test to find out the answer. (5p)

	Blue eye color	Other eye color
Blond hair color	64	17
Other hair color	50	182

8. At mid summers eve, 8 randomly selected children from the village Abborrviken and 8 other randomly selected children from the village Havsbyn were asked how many times they have gone for a swim outside during this summer. The data is given in the table below. Test a suitable hypothesis to find out if children from Abborrviken go swimming outside as often as children from Havsbyn before mid summer.

   It is not allowed to assume that data comes from the normal distribution. (5p)

Abborrviken	4	15	7	13	9	11	12	27
Havsbyn	6	0	1	8	3	10	2	14

GOOD LUCK!