UPPSALA UNIVERSITY
Department of Mathematics
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Exam in Mathematical Statistics
Inference 1 1MS035
2023–01–03

Time: 8.00-13.00. Limits for the credits 3, 4, 5 are 18, 25 and 32 points, respectively, including bonus points from hand-in assignments. The solutions should be well motivated.

Permitted aids: Pocket calculator. Dictionary. Formelsamling för stokastik.

1. We have a random sample 0, 1, 1, 2, 0, 0, 1, 2 from a discrete random variable X with probability function

$$
p_X(k) = \begin{cases} 2(1-\theta)/3, & k = 0, \\ (1+\theta)/3, & k = 1, \\ \theta/3, & k = 2, \\ 0, & \text{otherwise}, \end{cases}
$$

where $0 < \theta < 1$. Estimate θ using

  (a) the method of moments, (1p)

  (b) the least squares method, (1p)

  (c) maximum likelihood. (3p)

2. We have a random sample x_1, x_2, x_3, x_4, x_5 of the random variable X which has expectation $\mu + m$ and variance 2, a random sample y_1, y_2 of the random variable Y which has expectation $2m - \mu$ and variance 1, and a random sample z_1, z_2 of the random variable Z which has expectation m and variance 1. The means of the samples are denoted by $\bar{x}$, $\bar{y}$ and $\bar{z}$, respectively. We may assume that X, Y, Z are simultaneously independent.

  Two estimates of μ are proposed:

$$
\mu_1^* = \frac{\bar{x} - \bar{y} + \bar{z}}{2}, \quad \mu_2^* = \frac{5\bar{x} - 4\bar{y} + 3\bar{z}}{9}.
$$

  (a) Show that μ_1^* and μ_2^* are both unbiased for μ. (2p)

  (b) Which one of μ_1^* and μ_2^* is most efficient? Motivate your answer. (3p)

Please turn the page!

3. In his factory at the North Pole, Santa Claus wants to check if the production of baby dolls works as it should. He is satisfied if the average weight of all produced dolls, μ say, is greater than 0.5 kg.

He picks a random sample of 20 baby dolls, and measures the weight of all dolls in this sample. The average weight of the dolls in the sample is 0.54 kg.

Santa assumes that the weight of a doll is normally distributed with unknown expectation μ and (from previous studies) known standard deviation 0.1 kg.

(a) Perform a suitable hypothesis test to conclude if Santa can be satisfied. Choose the 5% level. (1p)

(b) Calculate the P value of the test in (a). (1p)

(c) Calculate the power of the test in (a) if $\mu = 0.56$. (2p)

(d) Santa is not satisfied with the power of the test, and regrets that he didn't take a bigger random sample. How many dolls should there be in the sample in order for the power for $\mu = 0.56$ to be at least 0.99? (1p)

4. At the North Pole factory, the pixies (tomtenissar) try out a new method to paint chess boards, which according to its inventor, the chief pixen Flirpo, is quicker than the old painting method. Five pixies, that are choosen randomly by Flirpo, get to paint two chess boards. Each of them uses the old method to paint one board and the new method to paint another. The painting skill may vary among pixies, but the skill is not assumed to increase with practice. (It is just the choice of method that might make a difference.)

The results (in seconds) are given in the table below. Assume that the painting times are independent and normally distributed.

Pixie	Flirpa	Florp	Firp	Furpie	Furp
Old method	2.34	1.45	5.67	0.67	1.58
New method	2.12	1.15	5.34	0.55	1.28

Test a suitable hypothesis to conclude if the new metod is better (quicker) than the old one. (5p)

Please turn the page!

5. A director of studies at the North Pole Technical University (NPTU) examines the starting salaries for newly examined students. She takes a random sample of 100 students. The mean monthly salary for these students is 23.5 North Pole Dollars (NPD) with sample standard deviation 2.5.

   It is not permitted to assume that salaries are normally distributed.

   (a) Calculate a 95% confidence interval for the expected salary of a newly examined student from NPTU. (4p)

   (b) Recently, the vice-chancellor of NPTU has claimed that the expected salary of a newly examined student from NPTU is 25 NPD. Given the result in (a), what can you say about this claim? (1p)

6. We have a random sample

$$4.2, \ 2.3, \ 0.4, \ 5.0, \ 3.7, \ 3.2, \ 0.7, \ 4.5, \ 2.6, \ 1.1,$$

   from $X \sim \text{Re}(0, \theta)$.

   Calculate a 95% confidence interval for θ. (5p)

   Hint: Start by considering the estimator $\max_{1 \leq i \leq 10} X_i$, where $X_1, ..., X_{10}$ are random variables corresponding to the random sample of observations.

7. A General Society Survey in the US cross-classified degree of fundamentalism of religious beliefs by the highest degree of education according to the table below.

   Are religious beliefs independent of education? Perform a suitable hypothesis test to find out. (5p)

	Religious Beliefs		
Education	Fundamentalist	Moderate	Liberal
High school/Junior college or lower	748	786	550
Bachelor or graduate	138	252	252

8. Apartments of the same size are sold in two parts of a city, East and West. Two random samples are taken, one for each part. For these samples, the prices in millions of Swedish krones are given in the following table.

East	6.6	11.0	5.3	7.2	5.8
West	7.6	7.9	13.0		

   It is not reasonable to assume that apartment prices are normally distributed.

   Are the prices for this kind of apartments equally distributed for East and West? Try to answer this question by performing a suitable hypothesis test.

   (5p)

GOOD LUCK!