UPPSALA UNIVERSITY
Department of Mathematics
Rolf Larsson

Exam in Mathematical Statistics
Inference 1 1MS035
2023–06–15

Time: 8.00-13.00. Limits for the credits 3, 4, 5 are 18, 25 and 32 points, respectively. The solutions should be well motivated.

Permitted aids: Pocket calculator. Dictionary. Formelsamling för stokastik.

1. We have a random sample 0.3, 0.7, 1.5, 0.8, 0.6 from a continuous random variable X with density function

$$f(x) = \frac{1}{2\theta^3} x^2 e^{-x/\theta},$$

where $x > 0$ and $\theta > 0$.

   (a) Estimate θ by using the method of moments. (1p)
   Hint: Without proof, you may use that $E(X) = 3\theta$.

   (b) Estimate θ by using the method of maximum likelihood. (4p)

2. We have a random sample x_1, x_2, x_3 of the random variable X which has expectation μ and variance 1, and a random sample y_1, y_2, y_3, y_4 of the random variable Y which has expectation 2μ and variance 4. The means of the samples are denoted by $\bar{x}$ and $\bar{y}$, respectively. We may assume that X and Y are independent.

   Two estimates of μ are proposed:

$$\mu_1^* = \frac{2\bar{x} + \bar{y}}{4}, \quad \mu_2^* = \frac{3\bar{x} + 2\bar{y}}{7}.$$

   (a) Show that μ_1^* and μ_2^* are both unbiased for μ. (2p)

   (b) Which one of μ_1^* and μ_2^* is most efficient? Motivate your answer. (3p)

Please turn the page!

3. The number of power failures at Donald's summer house follows a Poisson distribution with parameter (mean) λ. During one summer, the Larsson family rents Donald's summer house. Donald tells the family that λ is at most 1.

   (a) The family suspects that there are more power failures at the summer house than what Donald claims. In fact, it turns out that during the summer when they rent it, there are three power failures in total.

   Test a suitable hypothesis to try to check if Donald is telling the truth. (2p)

   (b) Calculate the power of the test in (a) if in fact, $\lambda = 5$. (3p)

4. Seasonal ranges (in hectares) for alligators were monitored by biologists on a lake in Florida. Five alligators monitored in the spring showed ranges of 8.0, 12.1, 8.1, 18.2, 31.7. Four different alligators monitored in the summer showed ranges of 102.0, 81.7, 54.7, 50.7.

   Estimate the difference between mean spring and summer ranges, with a 95% confidence interval. Be careful to state your assumptions. (5p)

5. We have one observation $x = 1.2$ of the random variable X, which is exponentially distributed with parameter β, i.e. it has density function $f(x) = \beta e^{-\beta x}$ for $x > 0$ and 0 otherwise.

   Calculate a 90% confidence interval for β. (5p)

6. Reaction times were measured for a random sample $x_1, ..., x_n$ of $n = 200$ car drivers. The mean reaction time in the sample was $\bar{x} = 1.1$ seconds, and the standard deviation was $s = 0.3$.

   (a) Calculate a 95% confidence interval for the mean reaction time for car drivers in the whole population. (4p)

   (b) Somebody claims that the mean reaction time for the whole population of car drivers is 1.0 seconds. Can you find support for this claim? (1p)

7. At the University of Falnarp, a genetic experiment was conducted. A particular type of beans were cultivated. Upon harvest, it was expected that four genetic variants, numbered 1,2,3,4, should occur according to the proportions 1:3:3:9 (i.e. the first variant occurs in 1/16 of the cases, the second variant occurs in 3/16 of the cases etcetera). The results of the experiment are given in the table below.

Variant	1	2	3	4
Frequency	6	21	30	71

   Does the experiment confirm the expectations about proportions? Try to answer this question by performing a suitable hypothesis test. (5p)

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8. The table below gives the total consumption of nuclear power electricity (in thousands of Tera Joule) in Sweden during the years 2000-2021 (data from Statistics Sweden, SCB). Is there any trend in this material? Check this by performing a suitable hypothesis test.

   It is not allowed to assume that the numbers are normally distributed. (5p)

Year	Consumption
2000	597
2001	771
2002	723
2003	717
2004	818
2005	755
2006	683
2007	662
2008	673
2009	540
2010	599
2011	603
2012	647
2013	662
2014	650
2015	559
2016	640
2017	643
2018	683
2019	682
2020	497
2021	541

GOOD LUCK!