UPPSALA UNIVERSITY
Department of Mathematics
Rolf Larsson

Exam in Mathematical Statistics
Inference 1 1MS035
2023–08–23

Time: 14.00-19.00. Limits for the credits 3, 4, 5 are 18, 25 and 32 points, respectively. The solutions should be well motivated.

Permitted aids: Pocket calculator. Dictionary. Formelsamling för stokastik.

1. Let X be a discrete random variable with probability function

$$p_X(x) = \begin{cases} 9\theta^2 & \text{if } x = 2, \\ 6\theta(1-2\theta) & \text{if } x = 4, \\ (1-3\theta)^2 & \text{if } x = 6, \\ 0 & \text{otherwise,} \end{cases}$$

   where $0 \leq \theta \leq 1/3$.

   We have a random sample $x_1 = 4$, $x_2 = 2$, $x_3 = 6$ from X.

   (a) Find the moment estimate of θ. (1p)

   (b) Find the maximum likelihood estimate of θ. (4p)

2. We have a random sample x_1, x_2, x_3, x_4 from the random variable X which has expectation 2μ and variance 2, and a random sample y_1, y_2 from the random variable Y which has expectation μ and variance 1. The means of the samples are denoted by $\bar{x}$ and $\bar{y}$, respectively. We may assume that X and Y are independent.

   Two estimates of μ are proposed:

$$\mu_1^* = \bar{x} - \bar{y}, \quad \mu_2^* = \frac{\bar{x} + \bar{y}}{3}.$$

   (a) Show that μ_1^* and μ_2^* are both unbiased for μ. (2p)

   (b) Which one of μ_1^* and μ_2^* is most efficient? Motivate your answer. (3p)

Please turn the page!

3. Following the so called "Ling's law", the time in days (not necessarily an integer) after the last of June until it is possible to pick lingonberries in the forest may, to a good approximation, be described by a random variable X with cumulative distribution function

$$F(x) = 1 - \exp\left(-\frac{\theta x^2}{10000}\right),$$

where $x > 0$, and otherwise, $F(x) = 0$. The parameter θ is unknown.

   (a) Linga believes that $\theta = 9$. Her friend Lingberth thinks that $\theta < 9$. One year, it is possible to pick lingonberries in the forest 60 days after the last of June. Is Lingberth right? Try to find out by testing a suitable hypothesis. Use the significance level 5%. (2p)

   Hint: A smaller θ corresponds to a larger X.

   (b) What is the power of the test in (a) for $\theta = 2$? (3p)

4. Ellen prepares for a big party for vegans. At the supermarket CAI, she buys 10 cauiliflowers (Swedish: blomkål). This sample of cauiliflowers has average weight 520 grams and standard deviation 46.5.

   Her friend Lena wants to help out. She goes to the supermarket POOC and buys 8 cauliflowers. This sample of cauiliflowers has average weight 560 grams and standard deviation 53.2.

   We may assume that the weight of a cauliflower is normally distributed. Let the expected weight of a cauliflower from CAI be μ_1, and correspondingly, the expected weight of a cauliflower from POOC is μ_2.

   (a) Calculate a 95% confidence interval for $\mu_1 - \mu_2$. Be careful to state all your assumptions. (4p)

   (b) Is there evidence that the expected weights of cauliflowers from CAI are different from the expected weights of cauliflowers from POOC? (1p)

5. In the male elite division of bandy in Sweden 2021-2022, 2158 goals were scored in 240 matches. Assume that the number of goals in a match is Poisson distributed, and that the numbers of goals in different matches are independent and identically distributed.

   (a) Calculate a 99% confidence interval for the expected number of goals in a random match. (3p)

   (b) Calculate a 99% confidence interval for the expected time in minutes between two goals. (Assume that every match takes exactly 90 minutes.) (2p)

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6. A machine chops boards (Swedish: brädor) in pieces of θ centimeters (not necessarily integer). After each board has been chopped, there is a remaining piece of board with a length that can be assumed to be uniformly distributed on the interval $(0, \theta)$.

   The number θ is unknown. One day, there were 120 remaining pieces with mean length 37.4 centimeters.

   (a) Estimate θ by using the method of moments. (1p)

   (b) Calculate a 95% confidence interval for θ. (4p)

7. The ice cream company SAI has the following slogan:
   Our ice cream is the best, pretty tasteless is the rest!

   During the summer, a random sample of 200 people tried chocolate ice cream of two brands, BG and SAI. Out of these 200 people, 112 said that the SAI chocolate ice cream tasted better than the BG chocolate ice cream, and the remaining 88 said the opposite (BG tasted better than SAI).

   Does chocolate ice cream from SAI taste better than chocolate ice cream from BG? Perform a suitable hypothesis test to find out. (5p)

8. After summer, a school class meets to compare their salaries in SKr per hour for their respective summer jobs. A random selection of four boys and four girls recieved hourly salaries according to the table below.

   Is there evidence that boys and girls recieve different hourly salaries at summer jobs? Perform a suitable hypothesis test to find out.

   It is not allowed to assume that the data is normally distributed. (5p)

Girls	102	97	98	130
Boys	100	90	75	92

GOOD LUCK!