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Matematiska institutionen
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Prov i matematik
KandMa2, Frist
KOMPLEX ANALYS 10hp
2015–01–14

Writing time: 14.00 – 19.00. Tools allowed: pens, pencils, rubber. Every correctly solved problem gives up to 5 points.

1. Solve the equation

$$\sin z - \cos z = i \ .$$

(The answer should be given in the form $a + bi$, where a and b are real.)

2. Find all functions $f = u + iv$ which are analytic in $\mathbb{C}$ and such that $xu(x, y)$ is the real part of an analytic function. The answer should be given as an expression in the variable $z = x + iy$.

3. Find a Möbius transformation which maps the disc $|z - 2| < 2$ onto the unit disc $|z| < 1$, maps the point 0 to the point 1 and maps the point 1 to the point $\frac{1}{2}i$.

4. Assume that γ is the positively oriented unit circle $|z| = 1$ in $\mathbb{C}$. Let

$$f(z) = \int_\gamma \frac{1}{\cos(\zeta)(\zeta - z)^3} \, d\zeta \ .$$

Find $f'(\frac{\pi}{4})$. (The answer should be given in the form $a + bi$ with $a, b \in \mathbb{R}$.)

5. Calculate the value of the integral

$$\int_{-\infty}^{\infty} \frac{x \sin 2x}{x^4 + 4} \, dx \ .$$

6. Determine the number of zeros of the polynomial $f(z) = z^6 - 9z^2 + 11$ in the annulus $\{z : 1 < |z| < 2\}$.

7. Find coefficients c_{-1} and c_1 in the Laurent series

$$\frac{1}{1 - e^z} = \sum_{n=-\infty}^{n=\infty} c_n z^n$$

convergent in the region $2\pi < |z| < 4\pi$.

(Continued on the next page!)

8. Assume that the functions f and g are analytic in the whole complex plane $\mathbb{C}$ and that $|f(z)| \leq |g(z)|$ for all $z \in \mathbb{C}$. Show that there exists a complex number α such that $f(z) = \alpha g(z)$ for all z.

Good Luck!

Svar till tentamen i KOMPLEX ANALYS 10hp 2015–01–14

1. $z_n' = \frac{\pi}{4} + 2\pi n - i \ln\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$ and $z_n'' = \frac{5\pi}{4} + 2\pi n - i \ln\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right), \quad n \in \mathbb{Z}$.

2. $f(z) = aiz + B, \quad a \in \mathbb{R}, B \in \mathbb{C}$.

3. $F(z) = \dfrac{z(2+3i) - 2i}{z(2-2i) - 2i}$.

4. $f'(\frac{\pi}{4}) = i\,\pi\, 11\sqrt{2}$.

5. $I = \dfrac{\pi\, e^{-2}}{2} \sin(2)$.

6. 6 zeros.

7. $c_{-1} = -3, \quad c_1 = \dfrac{1}{2\pi^2} - \dfrac{1}{12}$.