UPPSALA UNIVERSITET
Matematiska institutionen
Jörgen Östensson

Prov i matematik
KandFy3, KandMa2 m.fl.
Complex Analysis
2022–04–14

Writing time: 08.00 – 13.00. Allowed aids: Writing materials and a copy of the textbook Complex Analysis by Gamelin. Each problem has a maximum credit of 5 points. For the grades 3, 4 and 5 respectively, one should obtain at least 18, 25 and 32 points, respectively. Solutions should be clearly written and properly explained.

1. Solve the equation

$$\tan z = 2i.$$

The answer should be given in the form $a + ib$, where a and b are real numbers.

2. Find all functions $f = u + iv$ which are analytic in $\mathbb{C}$ and satisfy

$$\frac{\partial v}{\partial x} = u.$$

The answer should be given as an expression in the variable $z = x + iy$.

3. Determine the image of the region $\{z : \operatorname{Re} z > 0, -\frac{\pi}{2} < \operatorname{Im} z < \frac{\pi}{2}\}$ under the mapping $z \mapsto \left(\dfrac{e^z + i}{e^z - i}\right)^2$.

4. Find the Laurent series expansion centered at $z_0 = -1$ of the function

$$f(z) = \frac{1}{z^2(z-1)}$$

which is convergent in the annulus $1 < |z+1| < 2$.

5. Let γ denote the positively oriented unit circle $|z| = 1$ in $\mathbb{C}$. Put

$$f(z) = \int_\gamma \frac{1}{\cos(\zeta)(\zeta - z)^2}\, d\zeta.$$

Determine $f'(0)$.

6. Use the residue theorem to calculate the integral

$$\int_{-\infty}^{\infty} \frac{x \sin 3x}{x^4 + 4}\, dx.$$

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7. Determine the number of zeros of the polynomial

$$p(z) = z^5 - 2z^3 + 5z + 1$$

in the region $1 < |z| < 2$.

8. Find all entire functions f such that

$$|f(z)| \leq |z|^2 \quad \text{for all } z \in \mathbb{C}.$$

GOOD LUCK!