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1.) (a) $\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N_2(0, I_2)$

(b) $\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \Big| \begin{pmatrix} X_3 \\ X_4 \end{pmatrix} \sim N_2\left(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}\left(\begin{pmatrix} X_3 \\ X_4 \end{pmatrix} - \mu_2\right)\right), \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})$

$\Sigma_{12} = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & 0 \end{pmatrix}, \quad \Sigma_{22} = \begin{pmatrix} 2 & \frac{1}{2} \\ \frac{1}{2} & 2 \end{pmatrix}$

$\Sigma_{22}^{-1} = \frac{1}{4 - \frac{1}{4}} \begin{pmatrix} 2 & -\frac{1}{2} \\ -\frac{1}{2} & 2 \end{pmatrix} = \frac{4}{15} \begin{pmatrix} 2 & -\frac{1}{2} \\ -\frac{1}{2} & 2 \end{pmatrix}$

$\mu = E\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Big| \begin{matrix} x_3 \\ x_4 \end{matrix}\right) = \begin{pmatrix} \frac{2}{15}x_3 - \frac{1}{30}x_4 - \frac{3}{8} \\ 0 \end{pmatrix}$

$\operatorname{cov}\left(\begin{matrix} x_1 \\ x_2 \end{matrix} \Big| \begin{matrix} x_3 \\ x_4 \end{matrix}\right) \overset{?}{=} \begin{pmatrix} \frac{29}{30} & 0 \\ 0 & 1 \end{pmatrix}$

$MM = \Sigma_{12}\Sigma_{22}^{-1} = \begin{pmatrix} 0.13\overline{3} & -0.03\ldots \\ 0 & 0 \end{pmatrix} \overset{?}{=} \begin{pmatrix} \frac{1}{30} & \\ 0 & 0 \end{pmatrix} \begin{pmatrix} X_3 - 1 \\ X_4 - 1 \end{pmatrix}$

$\Rightarrow \mu = MM \begin{pmatrix} X_3 - 1 \\ X_4 - 1 \end{pmatrix}$

$\mu = \begin{pmatrix} 0.13\overline{3}(X_3 - 1) - 0.0\overline{3}(X_4 - 1) \\ 0 \end{pmatrix}$

$-0.13\overline{3} + 0.0\overline{3}$

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c) $E(X_3 | X_2)$

$\begin{pmatrix} X_3 \\ X_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 & \frac{1}{4} \\ \frac{1}{4} & 1 \end{pmatrix}\right)$

$EX_3 | X_2 = \mu_3 - \frac{1}{4}(X_2 - 0)$

$= 1 - \frac{1}{4}X_2$

d) $3\underbrace{E(X_2 | \cancel{X_3}, X_4)}_{= E(X_2 | X_4)} + 2$

$$\begin{pmatrix} X_2 \\ X_4 \end{pmatrix} \sim N_2\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 2 \end{pmatrix}\right)$$

$$X_2 | X_4 \sim N\left(0 + \frac{1}{2} \cdot \frac{1}{2} (X_4 - EX_4), \; \dots\right)$$

$$E(X_2 | X_4) = \left(\frac{1}{4} X_4 - \frac{1}{2}\right)$$

$$3\,E(X_2 | X_3 X_4) + 2 = \frac{3}{4} X_4 + \frac{3}{2}$$

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(3) 2 sample problem.

(a) $H_0 : \mu_X = \mu_Y$

(b) $S_{pool} = \frac{1}{n+m-2}\left(\sum (X_i - \bar{X})(X_i - \bar{X})^T + \sum (Y_i - \bar{Y})(Y_i - \bar{Y})^T \right)$

$(n+m-2) \cdot S_{pool} \sim W_p(n+m-2, \Sigma)$

(c) $S_1 = \frac{1}{n-1} \sum (X_i - \bar{X})(X_i - \bar{X})^T$

$\sim W_p(n-1, \Sigma)$

(d) S_{11} defined as. $S_{pool} = \begin{pmatrix} \underset{2\times 2}{S_{11}} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$

$S_1 \, (n+m-2) \sim W_2(n+m-2, \Sigma_{11})$

(e) only the taste
that are the observations of
coffein concenter, bitter substances,
alternative only bitter.

$X_i = \begin{pmatrix} X_{(1)i} \\ X_{(2)i} \end{pmatrix}$ $T = (X_{(2)} - \bar{X}_{(2)})^T \left(\frac{S_{11}}{\left(\frac{1}{n}+\frac{1}{m}\right)}\right)^{-1} (\cancel{Y}_{(2)} - \bar{Y}_{(2)})$

b.w.

Hotellings Test:

$$N_p(0,\Sigma)^T W_p(m,\Sigma)^{-1} N_p(0,\Sigma) \sim F_{p,m-p}$$

$$T^2 = (\bar{X}_1 - \bar{X}_2 - \delta_0)^T \left(\left(\frac{1}{n_1} + \frac{1}{n_2}\right) S_{pool}\right)^{-1} (\bar{X}_1 - \bar{X}_2 - \delta_0)$$

$$H_0: \mu_1 - \mu_2 = \delta_0$$

$$S_{pool} = \frac{1}{n_1+n_2-2} \left(\sum^{n_1} (X_{1_i} - \bar{X}_1)(\quad)^T + \sum^{n_2} (X_{2_i} - \bar{X}_2)(\quad)^T \right)$$

$$\sim F_{2,\, \underbrace{n_1+n_2-2}-2}$$

(4) CCA

$$X_i = \begin{pmatrix} \text{opacity} \\ \text{Viscosity} \\ \text{coffein} \\ \text{bitter subst.} \end{pmatrix} = \begin{pmatrix} X^{(1)} \\ X^{(2)} \end{pmatrix}$$

physic

chemical

$p=2$

$q=2$

(a) canonical variates

$U = a^T X^{(1)}$, $V = b^T X^{(2)}$

First (U_1, V_1) max. $\cancel{\text{cor}}\,(X^{(1)}, X^{(2)})$

Second (U_2, V_2) " $\text{cov}(X^{(1)}, X^{(2)})$ under $\perp U_1$ $\perp V_1$

cannonical corellation.

$\cancel{\text{cor}}\,(U_1, V_1)$

and $\text{cov}(U_2, V_2)$

(b) ~~First pair.~~

Need the eigenvalue and eigenvector of

$$\begin{pmatrix} 0.4371 & 0.2178 \\ \ldots & 0.1096 \end{pmatrix}$$

Example 10.1 of the book contains calculat-

$(\rho_1^{*2}, \rho_2^{*2}) = (0.5458, 0.009)$

$e_1 = (0.89, 0.4466)$

$a_1 = \rho_{11}^{-1/2} e_1 = \begin{pmatrix} 0.85 \\ 0.276 \end{pmatrix}$ $\rho_1^* = 0.74$

$b_1 = \begin{pmatrix} 0.4 \\ 0.54 \end{pmatrix}$

(d) ρ_1^{*2} too small.

(e) R-command. res.cc < cc(X, Y)

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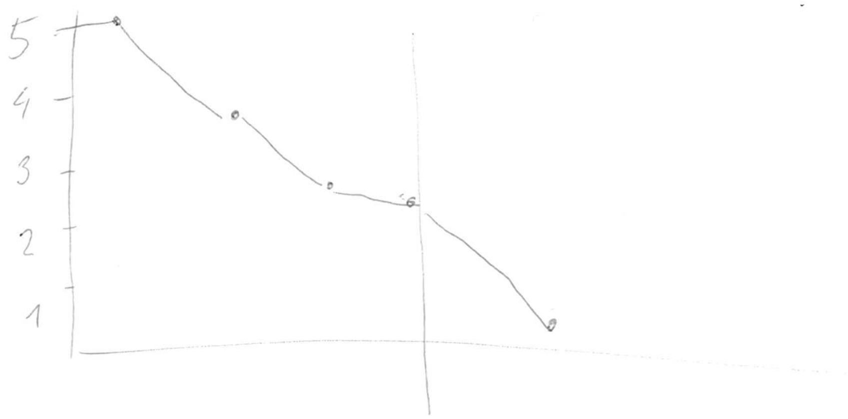
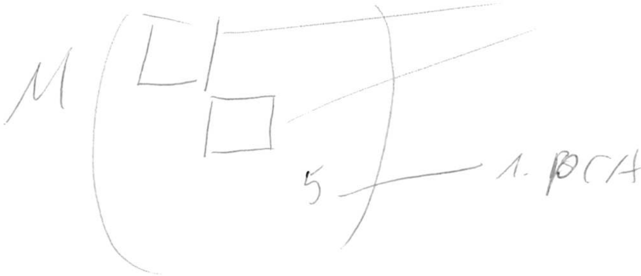
(5) PCA

eigenvalues 5, 3.5, 2.5, 1.9, 0.1

(a)



(b) Blockmatrix



we should
haver from each block
a represent.

e_1 First
e_2 second block
e_3 " "
e_4 Third blo

(c) $X_{(5)}$ | $0.7 X_3 + 0.7 X_4$

(d) $\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 5_1 \end{pmatrix}$

diagonal matrix
with the eigenvalues

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(6)

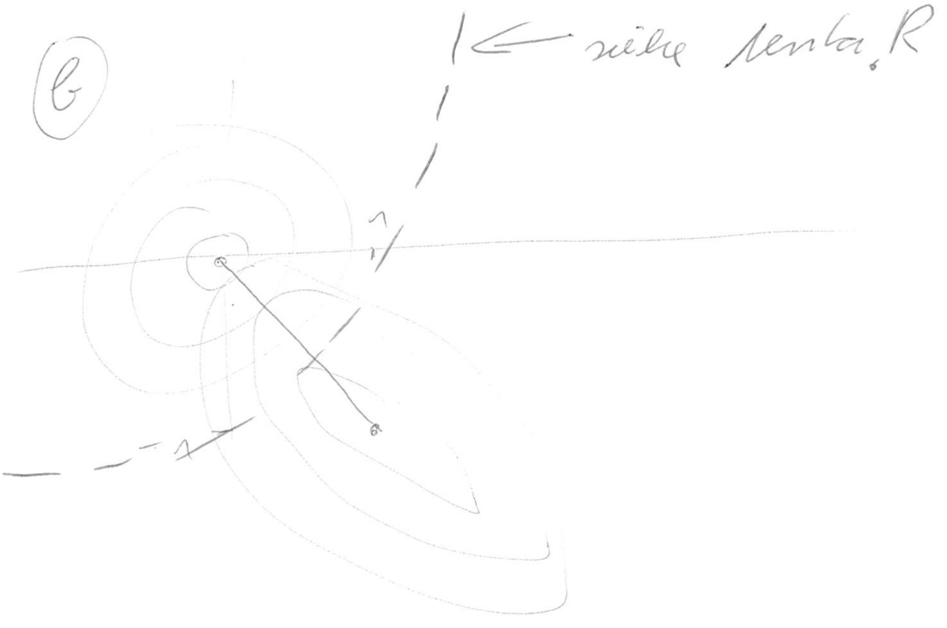
$$\Sigma = \begin{pmatrix} 1 & -0.8 \\ -0.8 & 1 \end{pmatrix}$$

(a) eigenvectors och eigenvalues

$\lambda: 1.8, 0.2$

$e_1 = \begin{pmatrix} -0.7 \\ 0.7 \end{pmatrix}, \quad e_2 = \begin{pmatrix} -0.7 \\ -0.7 \end{pmatrix} \quad = \frac{-1}{\sqrt{2}}$

(b)



(c) TPM

$= p_1 \, p(2|1) + p_2 \, p(1|2)$

$p_1 = p_2$

$p(1|2) = 2\,p(2|1)$

qda

$$R = \{ x : -\tfrac{1}{2} x^T (\Sigma_1^{-1} - \Sigma_2^{-1}) x + (\mu_1^T \Sigma_1^{-1} - \mu_2^T \Sigma_2^{-1}) x \geq \text{const} \}$$

$$\Sigma_2 = \begin{pmatrix} 1 & -0.8 \\ -0.8 & 1 \end{pmatrix} \qquad \Sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Sigma_1^{-1} > \Sigma_2^{-1} \ ?$$

$\Sigma_1 - \Sigma_2$ eigenvalues:

$$= \begin{pmatrix} 0 & 0.8 \\ 0.8 & 0 \end{pmatrix} : (0.8, -0.8)$$

$$-\frac{1}{2}x^T\underbrace{(\Sigma_1^{-1} - \Sigma_2^{-1})}_{\text{neg-def}} x + (\underbrace{\mu_1^T\Sigma_1^{-1}}_{0} - \mu_2^T\Sigma_2^{-1})x \geq c$$

$$\Sigma_2^{-1} = \begin{matrix} 2.7 & 2.2.. \\ 2.2.. & 2.77.. \end{matrix} \qquad \mu_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 \\ -1 \end{pmatrix}^T \Sigma_2^{-1}}_{} \quad \begin{pmatrix} 0.55.56 \\ -0.5.5 \end{pmatrix}$$

≥ 0!

$$\frac{1}{2}x^T\begin{pmatrix} 1.7 & 2.2 \\ 2.2 & 1.7 \end{pmatrix}x + \begin{pmatrix} 0.5..56 \\ -0....56 \end{pmatrix}^T \cancel{x} \geq c$$

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(7)

Similarity relation

distance

$d(x,y) = d(y,x)$

$d(x,y) > 0 \quad x \neq y$

1.25 $\quad d(x,y) = 0 \quad x \neq y$

$d(x,y) \leq d(x,z) + d(z,y)$

similarity

$s(x,y) = s(y,x)$

$s(x,y) > 0$

$s(x,y) = 1 \Rightarrow x = y$

$$s(x,y) = \frac{1}{1 + d(x,y)}$$

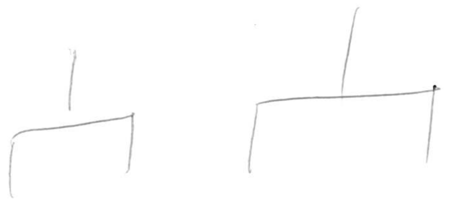
da gilt es keine Δ Ungleichung

besser im Test distances abfragen!

The same first letter = similar

$$\begin{array}{cccc} G & S & F & C \end{array}$$

$$\begin{pmatrix} 5 & & & \\ 3 & 5 & & \\ 0 & 0 & 5 & \\ 0 & 0 & 3 & 5 \end{pmatrix}$$



(8) 1MS003 11/1/21

(a) paired - sample. b) - - - -
Hotelling test.

(c) waves not normal
diff waves normal.
diff wave without outlier.
normal
but smaller p-value!

(d) Mnew
outlyer test

(e) no

(f) diff waves more relyable.

Mnew is not a good choise!