Multivariate

① $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \sim N_3\left(\begin{pmatrix} a \\ b \\ c \end{pmatrix}, \Sigma\right)$

$X \sim N(0,1) \Rightarrow a = 0$
$\sigma_{11} = 1$

$\begin{pmatrix} 1 & \frac{1}{2} & \\ \frac{1}{2} & \frac{1}{2} & \\ & & \end{pmatrix}$

$X|Y \sim N(y-1, \frac{1}{2})$

Formel
$X \sim N(a, \sigma_{11})$
$Y \sim N(b, \sigma_{22})$
$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left(\begin{pmatrix} a \\ b \end{pmatrix}, (\ )\right)$
$X|Y \sim N(a + \sigma_{12}\sigma_{22}^{-1}(y-b), \sigma_{11} - \sigma_{12}^2\sigma_{22}^{-1})$

$\sigma_{22} = \frac{1}{2}$

$\sigma_{21} = 0.5 = \frac{1}{2}$

$S = \frac{0.5 \cdot \sqrt{2}}{1}$

$b = 1$

$\sigma_{12}\sigma_{22}^{-1}(y-b) = y - 1$

$\sigma_{12}\sigma_{22}^{-1} = 1$

$\sigma_{12}\sigma_{22}^{-1} b = +1$

$\Rightarrow b = +1$

$\underbrace{\sigma_{11} - \sigma_{12}^2\sigma_{22}^{-1}}_{= 1} = 1 - \frac{1}{2}\frac{1}{2} \cdot 2$

$= \frac{1}{2}$

$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{8} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

$Y|Z = N(z-1, \frac{1}{4})$

$\sigma_{23}\sigma_{33}^{-1}(z-c) = z - 1$

$\sigma_{22} - \sigma_{23}^2\sigma_{33}^{-1} = \frac{1}{4}$

$\sigma_{23}\sigma_{33}^{-1} = 1$

$\frac{1}{2} \Rightarrow +\sigma_{23}^2\sigma_{33}^{-1} = +\frac{1}{4}$

$c = 1$

$\sigma_{23} = \sigma_{33}$

$\sigma_{23} = \frac{1}{4}$

$E(2x+4z-2\mu_1-4\mu_2)^2$
$= E(2x-2\mu)^2 + E(4z-4\mu_2)^2$
$+ 2\,E(2x-2\mu)(4z-4\mu_2)$

(1) $E(2x+4z) = \underline{\underline{2}}$

$Var(2x+4z) = 4\sigma_{11}^2 + 16\,\sigma_{33}^2 + 2\cdot 2\cdot 4\,\underbrace{Cov(x,z)}_{\frac{1}{8}}$

$= 4 + \frac{2\,\cancel{5}4\cdot 4}{4} + 2 = 10$

$2x+4z \sim N(2, 8)$

$c = Cov(Y, 2x+4z) = 2\,Cov(x,Y) + 4\,Cov(z,Y)$

$= 2\cdot\frac{1}{2} + 4\,\frac{1}{4} = 2$

$E(Y|2x+4z) = b + c\,s^{-2}(2x+4z-2)$

$= 1 + 2\,\frac{1}{8}\,(2x+4z-2)$

$= 1 + \frac{1}{4}(2x + \frac{1}{4}4z - \underbrace{\frac{1}{4}2}_{-\frac{1}{2}}$

$= \frac{1}{2}x + z + \frac{1}{2}$

$Var(Y|2x+4z) = \sigma_{22} - c^2 s^{-2}$

$= \left(\frac{1}{2}\right) - 2\cdot\cancel{2}^{1}\,\frac{1}{\cancel{10}_5} = \frac{1}{2} - \frac{2}{5}$

$= \frac{5-4}{10} = \underline{\frac{1}{10}}$

$(Y|2x+4z) \sim N(\frac{1}{2}x + z + \frac{1}{2}, \frac{1}{10})$

d) $E(Y|x) = b + \sigma_{12}\,\sigma_{22}^{-1}(x - a)$

$= 1 + \frac{1}{2}\cdot\left(\frac{1}{2}\right)^{-1} x = 1 + x$

e)

② MANOVA

$$\underset{p\times 1}{X_{ls}} = \mu + \alpha_s + \beta_l + \varepsilon_{ls} \quad \varepsilon_{ls} \sim N_p(0, \Sigma) \text{ i.i.d.}$$

$p = 6$

$i = 1 \dots n$ # of obs.

$\sum \alpha_s = \sum \beta_l = 0$

10 of each location $l = 1 \dots 10$

3 of each specy $s = 1 \dots 3$

$n = 30$

(a) $H_0: \beta_l = 0 \ \forall l$

(b)

Total: $\sum_{s}\sum_{l=1}(X_{ls} - \bar{X})(\ )^T =$

Factor 1: $\sum\sum (\bar{X}_{\cdot l} - \bar{X})(\ )^T$

Factor 2: $+ \sum\sum (\bar{X}_{s\cdot} - \bar{X})(\ )^T$

$+ \sum\sum (X_{ls} - \bar{X}_{\cdot l} - \bar{X}_{\cdot s} + \bar{X})(\ )^T$

Residual

Under H_0

(c) $X_{ls} = \mu + \alpha_s + \varepsilon_{ls}$

$\sum \alpha_s = 0$

2

LRT

$$T = \frac{\max_{\mu, \Sigma\, H_0} L(\mu, \Sigma)}{\max_{\mu, \Sigma} L(\mu, \Sigma)} \leq 1$$

$$L(\mu, \Sigma) \propto \frac{1}{|\Sigma|} \exp\Big(-\frac{1}{2}\, tr\,(\underbrace{\Sigma(x_{ij}-\mu_k)(\quad)}_{\hat{\Sigma} \text{ und } \hat{\mu} \text{ in every model}})\Big)^{T} = \Sigma$$

$$T = \frac{\det(\hat{\Sigma})}{\det(\hat{\Sigma}_{H_0})} \leq 1$$

T klein $\rightsquigarrow$ Alternative

p-value $P(T < t_{obs})$

0.001 $\Rightarrow$ reject H_0

locations are important.

(3)

Classification:

1) Population: white oak
   " : red oak

2.) classify the 10 oaks.

Factor analysis:
only white ~~or~~ only red. ?

2 samples are different.

inside of each sample the

$X_1^{(1)} \dots X_{n_1}^{(1)}$ i.i.d. of $X^{(1)}$

$X_1^{(2)} \dots X_{n_2}^{(2)}$ of $X^{(2)}$

$X^{(1)} - \mu = LF + \varepsilon$

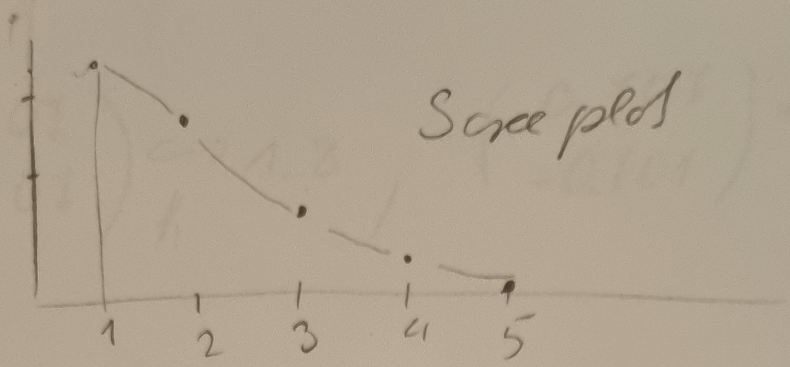
$\Sigma = LL' \approx$ mit hohen EV.

(R) da

factor

svd.

(4)

a) 

b) (2) and each group.

(c) $Y_{(i)} = e_i^T X$

$Y_1 = 0.43\, X_{(1)} + 0.65\, X_{(2)} + 0.61\, X_{(3)}$

$Y_2 = 0.7 \left(X_{(4)} + X_{(5)} \right)$

$Y_3 = 0.88\, X_{(1)} - 0.77\, X_{(2)} - 0.44\, X_{(3)}$

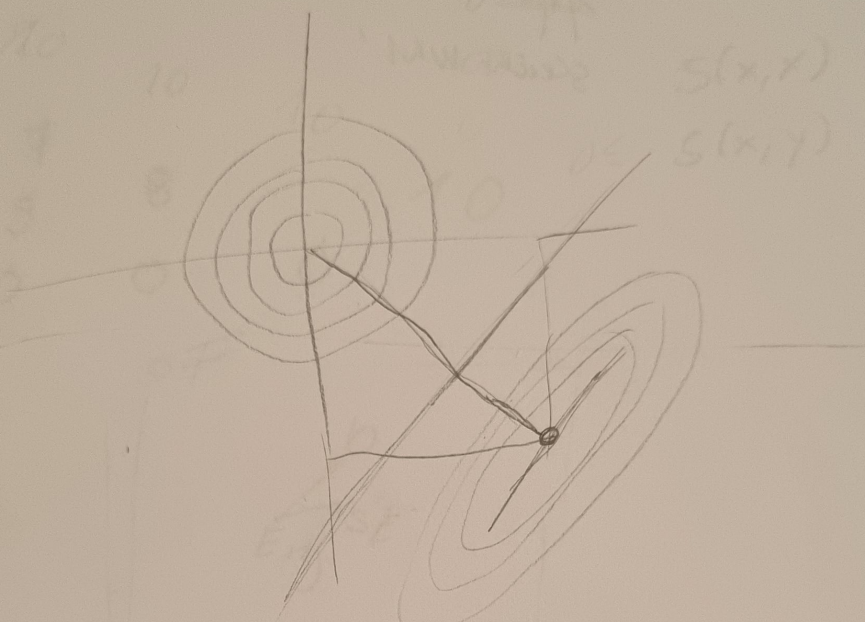
(d) $\mathrm{Cov}(Y) = \begin{pmatrix} 2.11 & & 0 \\ & 1.8 & \\ 0 & & 0.75 \end{pmatrix}$

---

(5)

a) $\begin{pmatrix} -0.707 \\ -0.707 \end{pmatrix} \underset{h}{\longleftrightarrow} 1.8$, $\begin{pmatrix} 0.707 \\ -0.701 \end{pmatrix} = 0.2$

b)



c) TPM Total prob of missclassi

$= p_1 \int_{R_2} f_1(x)\,dx + p_2 \int_{R_1} f_2(x)\,dx$ $\quad p_1 = p_2 = \frac{1}{2}$

optimal $R_2 = \{ x : f_2(x) > f_1(x) \}$

d) Best regions.

$\frac{1}{\sqrt{|\Sigma|}} \exp(-\frac{1}{2}(x-\mu)\Sigma^{-1}(x-\mu)) > f_1$

e)

6

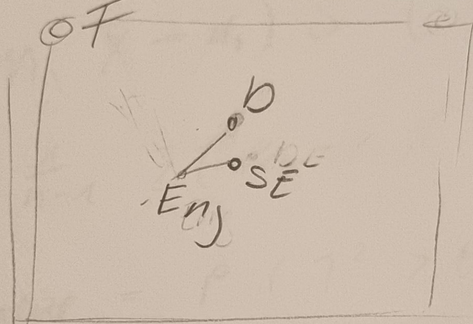
Ens De SE Fin

1 2

	Ens	De	SE	Fin
Ens	10			0
De	7	10		0
Se	5	8	10	0
Fin	0	0	0	10

$s(x,x)$ max

$0 \leq s(x,y) = s(y,x)$



---

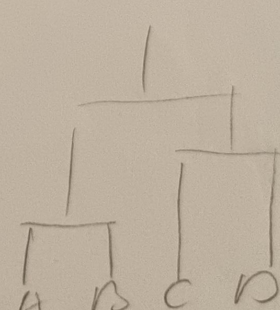
7 single linkage $d_{(u,v)w} = \min(d_{uw}, d_{vw})$

complete linkage $d_{(u,v)w} = \max(d_{uw}, d_{vw})$

(a) single l.:

(A B)

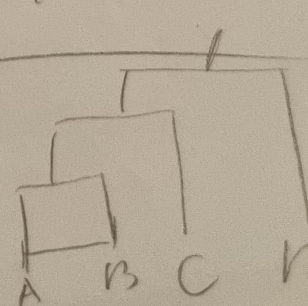
	AB	C	D
AB	10		
C	1	10	
D	1	4	10



(C D)

(e) (A B C)(D)

	AB	C	D
AB	10	10	
C	2		
D	5	4	10



8

a) $\underset{2\times1}{X_1} \dots \underset{2\times1}{X_n}$ i.i.d $n=45$

$\sim N_p(\mu, \Sigma)$

b) $H_0 : \mu = (200, 300)^T$

$H_0 \neq \mu = (193, 279)^T$

(c) $T^2 = n(\bar{X} - \mu_0)^T S^{-1} (\bar{X} - \mu_0)$

$S = \frac{1}{n-1} \Sigma (x_i - \bar{x})(\quad)^T$

$p\text{-value} = P(T^2 > t_{obs})$

d) yes.

(e) confidence regions contour sets.