$\boxed{1}$ (a) (i) - homogeneous!

(b) NO, NO, NO, YES

(c) $y' = t^2 y$

$y(-1) = 3$

$\boxed{2}$ Multiplying by $\mu(x) = x$, the equation becomes

$$(3x^2y + y^2x) + (x^3 + x^2y)\, y' = 0$$

which is exact. Its solution is

$x^3 y + \frac{1}{2} x^2 y^2 = c$ where c is arbitrary

$\boxed{3}$ (a) $y = c_1 e^t + c_2 t e^t$

(b) for example $e^t t \ln t$ (using variation of parameters)

$\boxed{4}$ (a) $y = c_1 e^t + c_2 e^{-t} + c_3$

(b) for example $y = -t^2/2$ (using method of undetermined coefficients)

(c) $y = c_1 e^t + c_2 e^{-t} + c_3 - \frac{t^2}{2}$

$\boxed{5}$ (a) regular singular point since $x p(x)$ and $x^2 q(x)$ are analytic at $x = 0$

(b) Indicial equation is $r^2 - \frac{1}{4} \Rightarrow r = \pm \frac{1}{2}$

We can find a Frobenius solution $y = \sum_{n=0}^{\infty} a_n x^{r+n}$ with $r = \frac{1}{2}$ (but not with $r = -\frac{1}{2}$!)

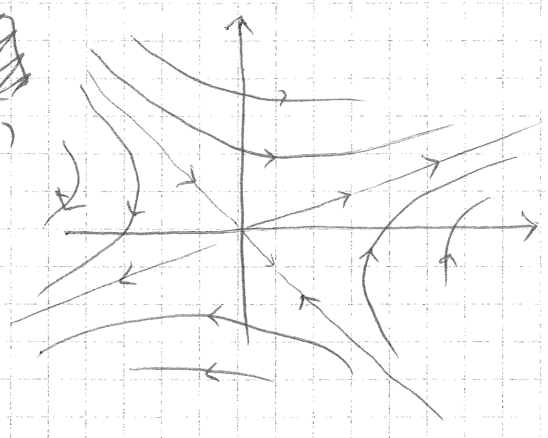
Plugging it into the equation we get a_0 can be arbitrary, $a_1 = 0$, $a_2 = -\frac{a_0}{2\cdot 3}$, $a_3 = 0$, ... $\left(a_{2n+1} = 0,\ a_{2n} = \frac{(-1)^n a_0}{(2n+1)!} \right)$

So, for example, $y(x) = x^{1/2} - \frac{1}{6} x^{5/2} + \frac{1}{5!} x^{9/2} - \dots$

$\boxed{6}$ (a) $\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \begin{pmatrix} 3 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$

(b) Saddle point ; unstable

(c)



$\boxed{7}$ (a) Plug it into the equations : $x = 0, y = 0$

(b) Both functions ~~are~~ ~~to~~ $F(x,y) = 3x - 2y - x^2$ and $G(x,y) = x + 21y^4 + x^{2017}$ are twice continuously differentiable, so the system is locally linear at every point, including $(0,0)$

(c) Linearized system at $(0,0)$ is

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

which has $(0,0)$ as an unstable node.

So the original non-linear system has also $(0,0)$ as an unstable node

$\boxed{8}$ Taking $V(x,y) = x^4 + 2y^2$ - positive definite, we have $\dot{V} = -2y^4$ is negative semi-definite. So by Lyapunov method, $(0,0)$ is a stable critical point