1MA032 2018-01-05

Problem 1 (a) Linear (i)

(b) False: for some cases, <u>some</u> solutions might be of the form te^{rt} (or, also, r may be complex)

(c) True (by the uniqueness theorem)

(d) $c_5 = a_0 b_5 + a_1 b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1 + a_5 b_0$

(e) ... if $p(x)$ and $q(x)$ are analytic at x_0

(f) $\begin{cases} e^{y^2+t^2} = y' \\ y(1) = 2018 \end{cases}$

(g) If geometric multiplicity is 2, then phase portrait is "radial nodal sink"

If geometric multiplicity is 1, then phase portrait is "improper nodal sink".

(h) Center point, spiral sink, spiral node.

Problem 2 (a)

solution: integrating factor $e^{-2\sin x}$ gives answer $y = 4e^{\sin x}(-1-\sin x) + ce^{2\sin x}$

(b) Just one by uniqueness theorem

(c) Zero: $y'(\pi/2) = 0$

Problem 3

(a) Char. eq.: $4r^2+4r+1=0$

Gen sol: $c_1 e^{-\frac{1}{2}t} + c_2 t e^{-\frac{1}{2}t}$

(b) Gen. sol: $\frac{1}{8}t^2 e^{-t/2} + \frac{1}{4}e^{t/2} + c_1 e^{-t/2} + c_2 t e^{-t/2}$

Problem 4

(a) Indicial eq.: $2r(r-1)+r-3=0$

Gen. sol: $c_1 x^{3/2} + c_2 x^{-1}$

(b) $y(x) = x^r \sum_{n=0}^{\infty} a_n x^n$,

where $r=\frac{3}{2}$ or $r=-1$;

if $r=\frac{3}{2}$: a_0 - arbitrary

$a_1 = 0$

$a_n = -\frac{(n-\frac{1}{2})}{n(2n+5)} a_{n-2}$

if $r=-1$: a_0 - arbitrary

$a_1 = 0$

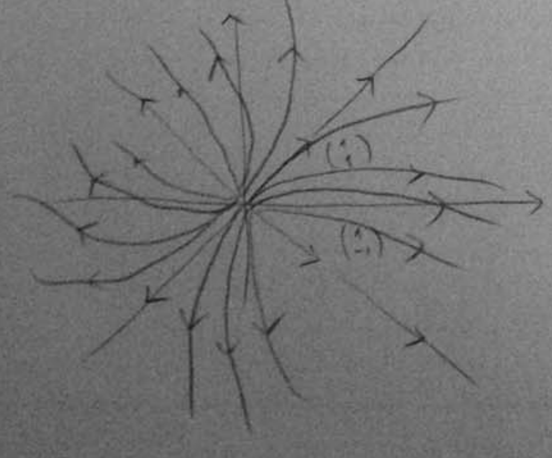
$a_n = -\frac{(n-3)}{n(2n-5)} a_{n-2}$

Problem 5 (a) $\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{3t}$

(b) Unstable

Proper nodal source

(c)



Problem 6

(a) $\lambda = 3i$ doesn't solve char. eq, so

$$\vec{Y}_1(t) = \begin{pmatrix} A \\ B \end{pmatrix} \sin 3t + \begin{pmatrix} C \\ D \end{pmatrix} \cos 3t$$

(b) $\lambda = 3$ solves char. eq, so

$$\vec{Y}_2(t) = \begin{pmatrix} A \\ B \end{pmatrix} t^3 e^{3t} + \begin{pmatrix} C \\ D \end{pmatrix} t^2 e^{3t} + \begin{pmatrix} E \\ F \end{pmatrix} t\, e^{3t} + \begin{pmatrix} G \\ H \end{pmatrix} e^{3t}$$

(c)

Gen. solution is $\vec{Y}_2(t) - \vec{Y}_1(t) + c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{3t}$

Problem 7

(a) $(0,0)$, $(0,2)$, $(3,0)$, $(1,1)$

(b) $(0,0)$ - unstable node

$(0,2)$ - as. stable node

$(3,0)$ - as stable node

$(1,1)$ - unstable saddle point

(c) In the region $x>0$, $y>0$ we have only one critical point and it is a saddle point. By the Poincare theorem, there is no periodic solution in $x>0$, $y>0$

Problem 8

(c) Using $V(x,y) = A x^k + B y^m$ with $k = 2018$, $m = 2$, $A = 1$, $B = -1009$,

we get that $\dot{V}$ is positive definite, and V has both positive and negative values around $(0,0)$. So by Liapunov theorem, $(0,0)$ is unstable

(b) Plug in $(0,0)$

(a) ... $V(0,0)=0$ and V has negative values around $(0,0)$ and $\dot{V}$ is negative definite

OR:

... $V(0,0)=0$ and V has positive values around $(0,0)$ and $\dot{V}$ is positive definite