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Matematiska institutionen
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Prov i matematik
1MA032
ODE I
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Allowed aids: writing materials. Each problem has a maximum credit of 5 points. For the grades 3, 4 and 5, respectively, one should obtain at least 18, 25 and 32 points, respectively. Solutions must be accompanied with explanatory text.

1. Solve the initial value problem

$$(\cos x)\, y' + (\sin x)\, y = 2 \sin x, \qquad y(0) = 1.$$

2. Find the general solution of the equation

$$y'' - y = 2e^x - x^2.$$

3. The function $y_1 = e^x$ is a solution of the equation

$$xy'' + (1 - 2x)y' + (x - 1)y = 0.$$

Find the general solution of the equation on the interval $(0, \infty)$.

4. Find one particular solution of the equation

$$8x^2y'' + 10xy' + (x - 1)y = 0.$$

in the form of an infinite series around 0. More specifically, find the first two non-zero terms of the series and the recurrence relation for the coefficients.

5. Find the general solution to the problem

$$\begin{aligned} x' &= x - y, \\ y' &= 6x - 4y, \end{aligned}$$

and sketch the phase portrait.

6. Prove that $(0, 0)$ is an asymptotically stable equilibrium point of the system

$$\begin{aligned} x' &= -3x^3 - y, \\ y' &= x^5 - 2y^3. \end{aligned}$$

7. Consider the system

$$\begin{aligned} x' &= y - x(1 - x^2 - y^2)^2, \\ y' &= -x - y(1 - x^2 - y^2)^2. \end{aligned}$$

Prove that $(0, 0)$ is a stable critical point. Prove that the system has periodic solutions.

8. Find the general solution to the system

$$\begin{aligned} x' &= x - 2y - z, \\ y' &= -x + y + z, \\ z' &= x - z. \end{aligned}$$

GOOD LUCK!

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