Regression Analysis 2023-05-29

Q1 (a) $\hat{\beta} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} = -1.66$

$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = 73$

(b) We use t-test. The error variance is estimated by $\hat{\sigma}^2 = 0.15$.

We then get $\hat{Cov}\begin{bmatrix}\hat{\alpha}\\ \hat{\beta}\end{bmatrix} = \hat{\sigma}^2 (X^T X)^{-1} = \begin{bmatrix} 0.225 & -0.015 \\ -0.015 & 0.0012 \end{bmatrix}$

Then the t value is $t = \frac{-1.66}{\sqrt{0.0012}} = -47.92$.

The critical value is 4.30. $t_{0.975}(2)$

We then reject H_0 $\beta = 0$.

(c) $73 - 1.66 \cdot 25 = 31.5$

(d) $31.5 \pm t_{0.975}(2) \sqrt{\hat{\sigma}^2 \left(1 + [1 \;\; 25](X^T X)^{-1}\begin{bmatrix}1\\25\end{bmatrix}\right)}$

Q2 (a) $\hat{\beta}_1 = \frac{-2y_1 + 0 + 2y_3}{8}$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}\bar{x} = \bar{y}$

(b) $Cov\begin{bmatrix}\hat{\beta}_0\\ \hat{\beta}_1\end{bmatrix} = \sigma^2 (X^T X)^{-1} = \sigma^2 \left(\begin{bmatrix}1 & -2\\ 1 & 0\\ 1 & 2\end{bmatrix}^T \begin{bmatrix}1 & -2\\ 1 & 0\\ 1 & 2\end{bmatrix}\right)^{-1}$

Q3. (a) GLS minimizes $(y - X\beta)^T \Lambda^{-1} (y - X\beta)$

The minimizer is $\hat{\beta} = (X^T \Lambda^{-1} X)^{-1} X^T \Lambda^{-1} y$

(b) $E(\hat{\beta}|X) = (X^T \Lambda^{-1} X)^{-1} X^T \Lambda^{-1} E(y|X) = \beta$ unbiased

$V(\hat{\beta}|X) = (X^T \Lambda^{-1} X)^{-1} X^T \Lambda^{-1} V(y|X) \Lambda^{-1} X (X^T \Lambda^{-1} X)^{-1}$

$= \sigma^2 (X^T \Lambda^{-1} X)^{-1}$

Q4 (a) $A^{**} = 475.5 / 3.45$

$B^{**} = -353 / 476.3$

$C^{**} = 332.9 \cdot (-0.394)$

$D^{**} = -1035 / (-2.162)$

(b) age, bmi, children, smokeyes, region southeast region southwest

(c) $256.9 \cdot 32 + 339.2 \cdot 26 + 475.5 \cdot 2 - 1035$

(d) $C^{**} < 0$ so the charge for female is higher

Charge for northwest is higher than other regions

Q5 (a) $A^{**} = 5.31821 / 2.944bo$

$B^{*} = -0.092bo / 0.04587$

(b) Yes, P value < 0.05

(c) $\log \mu = 5.31821 - 0.092bo \cdot Temp$

(d) $\exp(5.31821 - 0.092bo \cdot 31)$

(e) Hard to say because Temp is a continuous regressor